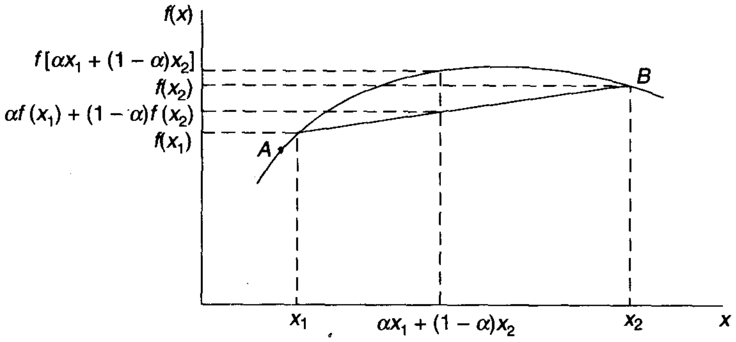

A function f(x) is strictly concave, if the line AB connecting any two points A and B on the function is completely below the function (curve AB).

LAGRANGE MULTIPLIERS

The Lagrange multipliers provide a powerful method for finding optima in multivariable problems involving functional constraints.

Function f(X) of with a Single Equality Constraints

Maximise or minimise f(X).

Subject to g(X) = 0

We shall write down the Lagranean of the function f(X) denoted by Lf(X, λ) and apply the Lagranean multiplier method.

$$Lf(X,\lambda) = f(X) - \lambda g(X) \text{L(f}$$

where λ is a Lagranean multiplier.

If more than one equality constraint is present in the problem.

$$Lf(X,\lambda) = f(X) - \lambda_1 g_1(X) - \lambda_2 g_2(X) - .... - \lambda_m g_m(X)$$

A necessary condition for the function to have a maximum or minimum is that the first partial derivatives of the function L should be zero.

$$\frac{dL}{dx_i} = 0, i = 1, 2, ...n$$

$$\frac{dL}{d\lambda_p} = 0, p = 1, 2....m$$

Now (n + m) simultaneous equations are solved to get a solution (X_0, λ_0).

Example

Maximise $f(X) = -x_1^2 - x_2^2$

Subject to $x_1 + x_2 = 4 \ \ or \ \ x_1 + x_2 - 4 = 0$

Solution

$$g(x) = x_1 + x_2 - 4 = 0$$

The Lagranean is

$$Lf(X, \lambda) = -x_1^2 - x_2^2 - \lambda(x_1 + x_2 - 4)$$

At the stationary point

$$\frac{\partial L}{\partial x_1} = -2x_1 - \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = -2x_2 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4) = 0$$

Solving these equations

$x_1 = 2$, $x_2 = 2$, $\lambda = -4$

Hence stationary point X = (2, 2) is a local maximum of f(X) and $f_{max}(X) = -8$.

Problem

Minimise $f(X) = 5x_1^2 + 2x_2 - x_1x_2$

Subject to $x_1 + x_2 = 3$

Answer: $x_1 = 5/2$, $x_2 = 31/12$, $f_{min}(X) = 357/72$

Function f(X) with Inequality Constraints

An equality constraint can be converted to an equality constraint by introducing an additional variable on the left hand side of constraint.

Thus a constraint $g(X) \le 0$ is converted as $g(X) + s^2 = 0$, where s^2 is a non negative variable. Similarly $g(X) \ge 0$ is converted as $g(X) - s^2 = 0$.

The solution is found by using Lagranean method.

When the Lagranean of f(X) is formed with either type of constraint, equating the partial derivative with respect to s gives

$$\lambda s = 0$$

which means that either $\lambda = 0$ or $s = 0$.

Kuhn Tucker Conditions

The conditions mentioned above lead to the statement of Kuhn-Tucker conditions. These conditions are necessary for a function f(X) to be a local maximum or a local minimum. The conditions for a maximising problem are given below:

Maximise f(X)