S'12:7FN:AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Discuss the continuity of the following function at $x = a$: **6**

$$f(x) = \begin{cases} (x-a)\sin(1/x-a), & \text{when } x \neq a \\ 0, & \text{when } x = a \end{cases}$$

(*b*) Discuss the convergence of the following series: **6**

$$\frac{1^2 \cdot 2^2}{\lfloor 1} + \frac{2^2 \cdot 3^2}{\lfloor 2} + \frac{3^2 \cdot 4^2}{\lfloor 3} + \cdots$$

(*c*) By using Taylor's theorem, show that

$$1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2}{2}e^x, \; x > 0.$$ **8**

(*Turn Over*)

2. (*a*) Find the relative maximum and minimum of the function

$$f(x, y) = 2(x^2 - y^2) - x^4 + y^4.$$ 6

(*b*) Using Cauchy mean value theorem, find the value of C for the following pair of functions :

$$f(x) = e^x \text{ and } g(x) = e^{-x}, x \in [a, b].$$ 6

(*c*) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 8

3. (*a*) Show that the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

is diagonalizable. Find P such that $P^{-1}AP$ is a diagonal matrix. 8

(*b*) Solve the following system of equations by Gauss elimination method : 6

$$2x + y - z = 4$$
$$x - y + 2z = -2$$
$$-x + 2y - z = 2$$

(*c*) Verify Stoke's theorem for the vector

$$\vec{v} = (3x - y)\vec{i} - 2yz^2\vec{j} - 2y^2z\vec{k}$$

where S is the surface of the sphere

$$x^2 + y^2 + z^2 = 16, z > 0.$$ 6

4. (*a*) Find the extreme values of

$$f(x, y, z) = 2x + 3y + z$$

such that $x^2 + y^2 = 5$ and $x + z = 1$. 8

(*b*) Evaluate the line integral of

$$\vec{v} = x^2\vec{i} - 2y\vec{j} + z^2\vec{k}$$

over the straight line from (–1, 2, 3) to (2, 3, 5). 6

(*c*) Find the eigenvalues and corresponding eigenvectors of the following matrix : 6

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

Group B

5. (*a*) Solve

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}.$$ 6

(*b*) Solve

$$y'' + 3y' + 2y = x + \cos x$$

by the method of variation of parameters. 8

(*c*) Find the Lagrange's interpolating polynomial that fits the following data values : 6

x	1	2	3
$f(x)$	1	7	61

6. (*a*) Determine the step size in an equidistant table for

$$f(x) = (1 + x)^6;\ x \in [0, 2]$$

if error in magnitude in linear interpolation is 5×10^{-5}. 6

(*b*) Evaluate the following integral by Simpson's one-third rule with $n = 4$:

$$\int_0^1 \frac{1}{(3+2x)} dx$$

Compare the result with the exact solution. 8

(*c*) For two events A and B, if $P(A) = 0{\cdot}5$, $P(B) = 0{\cdot}6$ and $P(A \cap B) = 0{\cdot}8$, then find the conditional probability $P(A/B)$ and $P(B/A)$. 6

7. (*a*) Find the mean variance and standard deviation of the distribution 6

X_i	2	3	8
$f(X_i)$	1/4	1/2	1/4

(*b*) Form a partial differential equation by eliminating the arbitrary function f and g from $z = y\,f(x) + x\,g(y)$. 6

(*c*) Solve

$$\frac{dx}{dt} + 2x - 3y = t$$

and

$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$ 8

8. (*a*) Solve

$$\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 x$$ 6

$$(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$$ 6

(*c*) Form a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0.$$ 8

Group C

9. Solve the following : 10 × 2

(*i*) If $f(x) = (x - 1)(x - 2)(x - 3)$, find c for $a = 0$, $b = 4$.

(*ii*) Find the first order partial derivative of the function

$$f(x, y) = x^4 - x^2y^2 + y^4$$

at (– 1, 1).

(*iii*) Find dy/dx from $f(x,y) = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - 1 = 0$.

(*iv*) Find the gradiant of $f(x, y) = y^2 - 4xy$ at (1, 2).

(*v*) Let $v_1 = (1, -1, 0)$, $v_2 = (0, 1, -1)$ and $v_3 = (0, 0, 1)$. Show that $\{v_1, v_2, v_3\}$ is linearly independent.

(*vi*) Solve

$$3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$$

(*vii*) Solve

$$(ax + hy + g)\, dx + (hx + by + f)\, dy = 0$$

(*viii*) Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

(*ix*) Find the divergence of the vector

$$\vec{v} = \left(x^2y^2 - z^3\right)\vec{i} + 2xyz\,\vec{j} + e^{xyz}\,\vec{k}$$

(*x*) If A, B, C are mutually exclusive and exhaustive events associated with a random experiment and $P(B) = 0{\cdot}6\, P(A)$ and $P(C) = 0{\cdot}2\, P(A)$, find $P(A)$.

W'12: 7 FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** and ALL *from* **Group C.**

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answer may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Let $y(x) = e^{4\sin^{-1}x}$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+4)y_n = 0.$$

Also, show that

$$(y_n)_{x=0} = \{(n-2)^2 + 4\}(y_{n-2})_{x=0}$$ 8

(*b*) Is the mean value theorem valid for $f(x) = x^2 + 3x + 2$ in the interval $1 \le x \le 2$? If the theorem is applicable, find a point C in $[1, 2]$. 6

(*c*) Use the appropriate test to examine whether the series

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{n}{2^n}+....$$

is convergent. 6

2. (*a*) Find the point at which the function $f(x)=(1/x)^x$ attains its extremum. Examine further that the maximum value of $f(x)$ is $(e)^{1/e}$. 5

(*b*) Let $u=\phi(y-z;\ z-x;\ x-y)$, then evaluate

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}.$$ 5

(*c*) Expand the function $f(x)=\tan^{-1}x$ in a finite series in powers of x, with remainder in the Lagrange's form. 5

(*d*) Determine the value of a such that divergence of

$$f=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$$

vanishes. 5

3. (*a*) Evaluate

$$\oint_C [(2x-y+4)\ dx+(5y+3x-6)\ dy]$$

around C: a triangle in the xy plane with vertices at (0, 0), (3, 0) and (3, 2) traversed in the counter-clockwise direction. 8

(*b*) Given $\vec{F}=(2x+3z)\hat{i}-(xz+y)\hat{j}+(y^2+2z)\hat{k}$, use Gauss's theorem to evaluate $\iint_S \vec{F}.\vec{n}\ dS$, where S is the surface of a sphere having centre at $(3,-1,-2)$ and radius 3. 6

(*c*) Evaluate $\iint_R \sqrt{x^2+y^2}\ dxdy$, where R is the region in the xy-plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$. 6

4. (*a*) Let A be a matrix such that

$$[1,0]A=[1,2,0]$$

and $$[0,1]A=[0,-1,1].$$

Then find the product matrix

$$A\begin{bmatrix}-1\\1\\1\end{bmatrix}$$ 8

(*b*) Find the matrix A representing the linear transformation that maps (x_1,x_2) on to $(2x_1-5x_2,\ 3x_1+4x_2)$ and verify the result. Does the inverse of this matrix A exists ? 6

(*c*) Of the three eigenvalues of the matrix

$$A=\begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix},$$

two are repeated. What are these eigenvalues ? Further, find the eigenvector corresponding to the non-repeated eigenvalue. 6

Group B

5. (*a*) Solve:

$$dy/dx=1-xy-y+x, \text{ given } y(0)=1.$$ 5

(*b*) Use the method of variation of parameters to solve

$$y''(x) + y(x) = \sec x.$$ 5

(*c*) Using the method of undetermined coefficient, solve the equation

$$y''(x) + 2y'(x) + y(x) = e^{-x}$$

subject to the conditions $y(0) = -1$ and $y'(0) = 1$. 10

6. (*a*) Eliminate the arbitrary functions *f* and *g* from the relation

$$z = f(x+10y) + g(x-10y)$$

to develop a partial differential equation of an appropriate order. Name the differential equation and its type. 6

(*b*) If the Laplace transform of a given function $f(t)$ is $F(s)$, then find the Laplace transform of the function

$$\{f(t-4)\ u(t-4)\},$$

where $u(t-a)$ represents the unit function with jump at $t = a$. 6

(*c*) Use the Laplace transform procedure to solve the initial value problem

$$\left.\begin{array}{l} y''(x) + 4y'(x) - 32y(x) = 0; \\ y(0) = 6, y'(0) = 0 \end{array}\right\}$$ 8

7. (*a*) State the Simpson's one-third rule and use it to evaluate

$$\int_0^6 \frac{dx}{1+x^2}.$$ 6

(*b*) Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial form and hence evaluate $\Delta^3 f(x)$. 6

(*c*) Derive the following :

(*i*) $\delta = E^{1/2} - E^{-1/2}$

(*ii*) $E = e^{hD}$

(*iii*) $D = (1/h)\ \log\ (1+\Delta)$.

where symbols have their usual meanings. 3 + 3 + 2

8. (*a*) A pointer moves along a fixed straight rod. Its distance *x* cm along the rod is given below for various values of the time *t* sec. Developing an appropriate difference table, find the velocity and the acceleration of the pointer : 10

t =	0	0.1	0.2	0.3	0.4	0.5	0.6
x =	30.13	31.62	32.87	33.64	33.95	33.81	33.24

(*b*) Compute the probability of obtaining at least two 'six' in rolling a fair die four times. 5

(*c*) Define the Normal distribution or the Gauss distribution in its standard form. Is the curve $f(x)$ in the defn. is symmetric with respect to $x = \mu$. If so, why ? What happens to the curve when $\mu = 0$? 5

Group C

9. Answer the following : 10 × 2

(*i*) Is the series $\sum_{r=1}^{\infty} (-1)^{r+1}$ convergent or divergent or oscillates finitely ?

(*ii*) Find the point at which the function

$$f(x) = a\sin^2 x + b\cos^2 x (a > b)$$

attains an extremum value.

(*iii*) Determine the region where the function $z = \log(x^2 + y^2 - 1)$ is defined.

(*iv*) . Find the rank of the matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

using the concept of 'linear independency'.

(*v*) The line integral

$$\int_{AB} [(y+z)dx + (z+x)dy + (x+y)dz]$$

is independent of the path from A to B. Is this statement *true* or *false* ?

(*vi*) Find the eigenvector corresponding to the eigenvalue 6 of the matrix

$$\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

(*vii*) Does there exists a function ϕ such that $\vec{F} = \nabla\phi$, where $\vec{F} = (2x-3)\hat{i} - z\hat{j} + \cos z\hat{k}$?

(*viii*) Find the inverse Laplace transform of

$$F(s) = 1/(s^2 + 100).$$

(*ix*) Name *any two* iterative methods by which one may solve a system of linear equations.

(*x*) Let the random variable X = number of heads in a single toss of a fair coin, has the possible values $X = 0$ and $X = 1$ with probabilities $P(X = 0) = 1/2$ and $P(X = 1) = 1/2$. Then find the mean and variance.

S'13 : 7FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $y(x) = \log\left\{x + \sqrt{1+x^2}\right\}$, then prove that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$$

Also, find out $(y_6)_{x=0}$ and $(y_7)_{x=0}$. **8**

(*b*) If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \cdots + \frac{a_{n-1}}{2} + a_n = 0$

then prove that the equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$$

has least one real root between 0 and 1. **6** (*c*) The curve $y^2(a+x) = x^2(3a-x)$ revolves about the axis of x. Find the volume generated by the loop. 6

2. (*a*) Discuss the convergence of the infinite series

$$\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\cdots ad \text{ inf.}$$ 5

(*b*) If $V = (x^2+y^2+z^2)^{-1/2}$, show that

$$x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}+z\frac{\partial V}{\partial z} = -V$$ 5

(*c*) Show that $\sin^p\theta \cos^q\theta$ attains a maximum value when $\theta = \tan^{-1}\sqrt{p/q}$. 5

(*d*) Find the unit normal to the surface $\phi(x,y,z) \equiv x^4 - 3xyz + z^2 = 0$ at the point $p(1, 1, 1)$. 5

3. (*a*) Evaluate

$$\iint_S \vec{F}\cdot\hat{n}\, ds,$$

where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and s is the surface of cylinder $x^2+y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 8

(*b*) Verify Green's theorem in plane for

$$\oint_C \{(xy+y^2)dx + x^2dy\}$$

where C is bounded by the curves $y = x$ and $y = x^2$. 6

(*c*) Expand $\log \sin x$ in powers of $(x-2)$ up to 4 terms containing cube of $(x-2)$. 6

4. (*a*) Show that the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies its characteristic equation. Does A^{-1} exist? If yes, find A^{-1} using Cayley-Hamilton theorem. 8

(*b*) Prove that two similar matrices of same order have same eigenvalues. 6

(*c*) If P_n denotes the space of all polynomials of degrees less than or equal to n, with real coefficients, then find the matrix of differential operator $T: P_4 \to P_4$ defined by

$$T(f(x)) = \frac{d}{dx}f(x)$$

under usual basis of P_4. 6

Group B

5. (*a*) Show that the equation

$$(4x+3y+1)dx + (3x+2y+1)dy = 0$$

represents a family of hyperbolas having as asymptotes the lines $x + y = 0$ and $2x + y + 1 = 0$. 8

(*b*) By method of variation of parameters, solve the differential equation

$$D^2y(x) - 2Dy(x) + y(x) = \frac{e^x}{2x}$$

where $D \equiv d/dx$. 6

(c) Obtain the Laplace transform of the function, $f(t)$, given by

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$ 6

6. (a) Solve

$$(x^2y^2 + xy + 1)\,y\,dx + (x^2y^2 - xy + 1)\,x\,dy = 0$$ 5

(b) Obtain a partial differential equation by eliminating the arbitrary function, f, from equation

$$x + y + z = f(x^2 + y^2 + z^2)$$ 5

(c) Solve the simultaneous differential equation

$$\frac{dx}{dt} + ax = y$$

$$\frac{dy}{dt} + ay = x$$

with initial conditions $x(0) = 0$, $y(0) = 1$ and using the laplace transforms. 5

(d) Solve the partial differential equation $p + q = pq$ where

$$p = \partial z/\partial x \text{ and } q = \partial z/\partial y.$$ 5

7. (a) The population of a town in the decennial census was as given below. Estimate the population for the year 1895. 6

Year : x	1891	1901	1911	1921	1931
Population : y (in thousand)	46	66	81	93	101

(b) State and derive trapezoidal rule for numerical integration and also derive the expression for error formula. 6

(c) From the following table of values of x and y, obtain dy/dx and d^2y/dx^2 for $x = 1{\cdot}2$: 8

x	1·0	1·2	1·4	1·6	1·8	2·0	2·2
y	2·7183	3·3201	4·0552	4·9530	6·0496	7·3891	9·0250

8. (a) Prove that if E and F are two independent events, then the events E and F' are also independent. 6

(b) Show that, in Poisson distribution with unit mean, mean deviation about mean is $(2/e)$ times the standard deviation. 6

(c) If the random variables X_1 and X_2 are independent and follow chi-square distribution with n d.f., show that $\sqrt{n}\,(X_1 - X_2)/2\sqrt{X_1X_2}$ is distributed as student's t with n d.f. independently of $X_1 + X_2$. 8

Group C

9. Answer the following : 10 × 2

(i) Evaluate

$$\int e^x(x\cos x + \sin x)dx.$$

(ii) Is the series

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

convergent ? Give reason for it.

(iii) Find $\text{div}\left(\frac{\vec{r}}{|\vec{r}|}\right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(*iv*) Show that

$$E \equiv 1 + \Delta \text{ and } \Delta \equiv \nabla(1 - \nabla)^{-1}.$$

(*v*) If

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

then find the sum and product of the eigenvalues of matrix A.

(*vi*) Change the order of integration in

$$\int_0^a \int_{mx}^{lx} f(xy)\,dy\,dx.$$

(*vii*) Find

$$\mathcal{L}^{-1}\left[\frac{s-4}{s^2 - 4s + 13}\right].$$

(*viii*) Under what condition the differential equation

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

will be exact and why ?

(*ix*) Write two basic characteristics of poisson distribution.

(*x*) Find length of the arc of semi-cubical parabola $ay^2 = x^3$ from the vertex to the point (a, a).

W'13: 7 FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Prove that

$$\frac{d^3y}{dx^3} = -\frac{\frac{d^3y}{dx^3}\cdot\frac{dy}{dx} - 3\left(\frac{d^2y}{dx^2}\right)^2}{\left(\frac{dy}{dx}\right)^5}$$ **8**

(*b*) Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. **8**

(*c*) If $\vec{f} = 3xy\vec{i} - y^2\vec{j}$, evaluate

$$\int_C \vec{F}\cdot d\vec{r}$$

where C is the curve in the (x, y) plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. **4**

2. (*a*) Apply Maclaurin's theorem to obtain the expansion of sec x. 6

(*b*) Verify Green's theorem for $f(x, y) = e^{-x} \sin y$, $g(x, y) = e^{-x} \cos y$ and C is the square with vertices at $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$ and $(0, \pi/2)$. 8

(*c*) Determine the rank of the matrix

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$ 6

3. (*a*) Show that the matrix A given by

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

is diagonalizable. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

(*b*) Discuss the convergence of the following series :

$$\sum x^n/3^n n^2, \text{ for } x > 0$$ 5

(*c*) Evaluate

$$\nabla e^{r^2}, \text{ where } r^2 = x^2 + y^2 + z^2.$$ 5

4. (*a*) Evaluate

$$\iint_A xy\,dy\,dx$$

where A is the domain bounded by x-axis, ordinate $x = 2a$, and the curve $x^2 = 4ay$. 8

(*b*) Prove that

$$\frac{x}{a} + \frac{y}{b} = 1$$

touches the curve $y = be^{-x/a}$ at the point where the curve crosses the x-axis. 6

(*c*) Show that for $0 < u < v$, $(v - u)/(1 + v^2) < \tan^{-1} v - \tan^{-1} u < (v - u)/(1 + u^2)$. 6

Group B

5. (*a*) Solve

$$(1 + y^2) + (x - e^{-\tan^{-1} y})\, dy/dx = 0$$ 5

(*b*) For the following given values of x and $\log_{10} x$

(300, 2·4771), (304, 2·4829), (305, 2·4843) and (307, 2·4871)

find the value of $\log_{10} 301$ using Lagrange's interpolation formula. 7

(*c*) An integer is chosen at random from 200 digits. What is the probability that the integer is divisible by 6 or 8 ? 8

6. (*a*) Find the general solution of the differential equation

$$(D^3 + 3D^2 + 2D)\, y = x^2$$ 6

(*b*) Find the Fourier transform of the function

$$f(t) = e^{-a|t|}, -a < t < \infty, a > 0$$

What is the inverse transform for it ? 7

(*c*) Let X be a normal variate with mean 30 and standard deviation 5. Find the probability that $26 \leq X \leq 40$. 7

7. (*a*) Evaluate the integral

$$\int_0^1 \frac{dx}{3+2x}$$

using Simpson's one-third rule with $n = 2, 4$. Compare the result with the exact solution. 7

(*b*) Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis. 6

(*c*) Apply the method of variation of parameters to solve

$$y_2 + a^2y = \text{cosec } ax. \qquad 7$$

8. (*a*) The integrating factor of the following equation is of the form y^n. Find n and solve the equation

$$y^{n+1}\sec^2x\, dx + \{3y^n \tan x - \sec^2 y\, y^{n-2}\}\, dy = 0 \qquad 8$$

(*b*) Solve the simultaneous differential equations

$$\frac{dx}{dt} - y = t$$

$$\frac{dy}{dt} + x = 1 \qquad 6$$

(*c*) Find the Laplace transform of the function

$$f(t) = \begin{cases} 0 & , \quad 0 \le t < 2, \\ K & , \quad t \ge 2 \end{cases}$$

and K is a constant. 6

Group C

9. Answer the following : 10 × 2

(*i*) Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{\sqrt{x}}; \quad y(2) = 4$$

(*ii*) Let A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = (3/2)\, P(A)$ and $P(C) = (1/2)\, P(B)$, the value of $P(A)$ is ____ .

(*iii*) A continuous random variable X has a p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. What will be the value of $P(X \le a)$?

(*iv*) Find the interval in which the equation

$$f(x) = x^6 - x - 1 = 0$$

has exactly one positive real root.

(*v*) If Δ and ∇ are the forward and backward difference operators respectively, then the value of $\Delta - \nabla$ is ____ .

(*vi*) Solve the differential equation

$$\frac{dy}{dx} + y\frac{d\phi}{dx} = \phi(x)\left(\frac{d\phi}{dx}\right)$$

(*vii*) State Rolle's theorem.

(*viii*) Discuss the applicability of Lagrange's mean value theorem for

$$f(x) = (1/x) \text{ ln } [-1,\ 1]$$

(*ix*) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^3$ at $(1, 2, 3)$ in the direction of $3\vec{i} + 4\vec{j} - 5\vec{k}$.

(*x*) Find the gradient of the following scalar field :

$$f(x, y) = y^2 - 4xy \text{ at } (1, 2).$$

S'14: 7FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $y = (x^2 - 1)^n$, prove that

$$(1 - x^2)\, y_{n+2} - 2xy_{n+1} + n\,(n + 1)\, y_n = 0$$

Hence, if $P_n = \frac{d^n}{dx^n}(x^2 - 1)^n$, show that

$$\frac{d}{dx}\left\{\left(1-x^2\right)\frac{dP_n}{dx}\right\} + n\,(n+1)P_n = 0.$$ 6

(*b*) Test for convergence the series

$$\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \cdots$$ 4

(*c*) Find the area of the surface formed by the revolution of $x^2 + 4y^2 = 16$ about its major axis. 6

(*d*) If $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$, evaluate

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$ 4

2. (*a*) Expand sin log $(x^2 + 2x + 1)$ in powers of x by Maclaurin's theorem as far as the term in x^4. 6

(*b*) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \cos nx \, dx$, prove that

$$(m + n) I_{m,n} = m I_{m-1,n-1}.$$

Hence, evaluate

$$\int_0^{\pi/2} \cos^3 x \cos 2x \, dx.$$ 6

(*c*) If $u\vec{F} = \nabla v$, where u and v are scalar field and $\vec{F}$ is a vector, show that $\vec{F} \cdot \text{curl } \vec{F} = 0$. 4

(*d*) Are the following vectors linearly independent :

$\vec{x}_1 = (1, 2, 4)$, $\vec{x}_2 = (2, -1, 3)$, $\vec{x}_3 = (0, 1, 2)$, $\vec{x}_4 = (-3, 7, 2)$? If so, find the relation between them. 4

3. (*a*) Expand $\log x$ in powers of $(x - 1)$ by Taylor's theorem and find the value of $\log 1{\cdot}1$. 5

(*b*) Find the work done in moving a particle once round the circle $x^2 + y^2 = 9$ in the xy-plane, if the field of force is given by

$$\vec{F} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$$ 5

(*c*) Given $x + y + z = a$, find the maximum value of $x^m y^n z^p$. 5

(*d*) Test for consistency and solve the following system of equations :

$$3x_1 - 6x_2 - x_3 - x_4 = 0, \; x_1 - 2x_2 + 5x_3 - 3x_4 = 0,$$
$$2x_1 - 4x_2 + 3x_3 - x_4 = 3.$$ 5

4. (*a*) Evaluate

$$\iint_R xy(x + y)\,dx\,dy,$$

where R is the area between $y = x^2$ and $y = x$. 4

(*b*) Use divergence theorem to evaluate

$$\iint_S (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$$

where S is the portion of the plane $x + 2y + 3z = 6$, which lies in the first octant. 4

(*c*) A linear transformation T on R^3 is given by

$T(x_1, x_2, x_3) = (-x_1 - x_2 + x_3, x_1 - 4x_2 + x_3, 2x_1 - 5x_2)$.

Determine the matrix representation of T with respect to the following ordered bases : 6

$B_1 = \{(1, 0, 0), (1, 1, 0), (-1, -1, 1)\}$,

$B_2 = \{(1, 0, 2), (2, 1, 0), (1, 0, 0)\}$

(*d*) Use Caley-Hamilton theorem to obtain the inverse of the following matrix : 6

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

Group B

5. (*a*) Integrate

$$\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$$

and obtain the cubic curve satisfying this equation and passing through the origin. 4

(*b*) Solve

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=xe^{3x}+\sin 2x$$ 6

(*c*) Solve the partial differential equation

$z^2 = 1 + p^2 + q^2$, $p = \partial z/\partial x$, $q = \partial z/\partial y$. 6

(*d*) Obtain the Laplace transform of the function, $f(t)$, given by

$$f(t)=\begin{cases} t/T, & 0<t<T \\ 1, & t>T \end{cases}$$ 4

6. (*a*) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$$ 6

(*b*) Find the Fourier transform of

$$f(x)=\begin{cases} 1, & |x|\le a \\ 0, & |x|>a \end{cases}$$ 4

(*c*) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2}-2\frac{dx}{dt}+2x=0;\ x=\frac{dx}{dt}=1$$

when $t = 0$. 4

(*d*) Following table is given :

x :	1.0	1.4	1.8	2.2	2.6
$f(x)$:	3.49	4.82	5.96	6.50	7.25

find $f(1.2)$ and $f(2.4)$. 3 + 3

7. (*a*) Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^2)\,dy = 0$. 5

(*b*) A curve is drawn to pass through the points given by the following table :

x :	1	1.5	2	2.5	3	3.5	4
y :	2	2.4	2.7	2.8	3	2.6	2.1

Using Simpson's one-third rule, estimate the area bounded by the curve, the x-axis and the lines $x = 1$, $x = 4$. 5

(*c*) An oil drilling company ventures into various locations and their success/failure is independent from one location to another. Suppose the probability of a success at any specific location is 0·25.

(*i*) What is the probability that a driller drills 10 locations and find 1 success ? 3

(*ii*) The driller feels that he will go bankrupt, if he drills 10 times before the first success occurs. What are the driller's prospects for bankruptcy ? 3

(*d*) The average life of a certain type of small motor is 10 years with a standard deviation of two years. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 3% of the motors that fail, how long a guarantee should he offer ? Assume that the life-time of a motor follows a normal distribution. Given : $\phi(-1{\cdot}881) = 0{\cdot}03$, where

$$\phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\,dx.$$ 4

8. (*a*) Solve, by the method of separation of variables,

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$$

where $u(x, 0) = 3e^{-5x} - 2e^{-3x}$. 5

(*b*) Use Lagrange's interpolation formula to find $f(x)$ from the following table :

x :	5	6	9	11
$f(x)$:	12	13	14	16

Hence, find $f(10)$. 6

(*c*) The average number of miles per litre of petrol of brand A found by running five similar cars under identical conditions was 6·6 and a standard deviation of 0·12. When brand B of petrol was used in six similar cars under conditions identical to the former, the average mileage came out 6·3 with a standard deviation of 0·13. Can you conclude, on the basis of this data, that brand A petrol gives significantly more mileage than the brand B ? (Given : $t = 0{\cdot}05$ for degree of freedom 9 is 2·26) 5

(*d*) Given $x = 4y + 5$, $y = kx + 4$ are the regression lines of x on y and y on x, respectively. Show that $0 \le k \le 1/4$. If $k = 1/8$, find the means of two variables and the coefficient of correlation between them. 4

Group C

9. Answer the following : 10 × 2

(*i*) The point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1, 0)$ and $B(2, 1)$, where the tangent is parallel to the chord AB is ____ .

(*ii*) The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{r^2}{n^3 + r^3}$ is ____ .

(*iii*) If $f(x, y) = \frac{1}{x^3} + \frac{1}{x^2 y} + \frac{1}{x^3 + 5y^3}$, then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 3f$ is ____ .

(*iv*) The partial differential equation corresponding to the function $z = f(x + ct) + g(x - ct)$ is ____ .

(*v*) The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ is ____ .

(*vi*) The value of $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$ free from Δ and E is ____ .

(*vii*) If $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 200, y_4 = 100$, then the value of $\Delta^4 y_0$ is ____ .

(*viii*) If $u = f(x - y, y - z, z - x)$, then $(\partial u/\partial x) + (\partial u/\partial y) + (\partial u/\partial z)$ is ____ .

(*ix*) If $f(x) = kx(1 - x)$, $0 \le x \le 1$ is the probability density function of a random variable x, then the value of k is ____ .

(*x*) If X is a Poisson variate such that $P(X = 1) = P(X = 2)$, then $P(X < 2)$ is ____ .

W'14:7FN:AN209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Give an example each of (*i*) a convergent series ; (*ii*) a divergent series ; and (*iii*) an oscillatory sources. 3

(*b*) Prove or disprove the statement that the infinite series

$$x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1.3}{2.4}\cdot\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\cdot\frac{x^7}{7}+\cdots$$

is convergent, if $-1<x<1$. 6

(*c*) If $y=\sin^{-1}x/\sqrt{1-x^2}$; $|x|<1$, then prove that $(1-x^2)y_2-3xy_1-y=0$. 4

(*d*) (*i*) Given the equation $2x^2-yz+xz^2=4$, evaluate $\partial x/\partial y$ and $\partial x/\partial z$. 3

(*ii*) If $u = x f(y/x) + g(y/x)$, then show that
$x(\partial u/\partial x) + y(\partial u/\partial y) = x f(y/x)$. 4

2. (*a*) Evaluate $\iint dx dy$ over the domain bounded by $y = x^2$ and $y^2 = x$. 5

(*b*) Let $\vec{\alpha}$ be a constant vector. Then prove the correctness or otherwise of the result $curl(\vec{r} \times \vec{\alpha}) = -2\vec{\alpha}$, where $\vec{r} = x\hat{i} + y\hat{i} + z\hat{k}$. 5

(*c*) Consider the integral

$$I_m = \int x^m e^x dx$$

and arrive at the reduction formula

$$I_m = x^m e^x - m I_{m-1}.$$

Hence, evaluate $\int_0^4 x^4 e^x dx$. 5

(*d*) Let a circle $x^2 + y^2 = a^2$ revolves round the x axis. Then find the surface area and volume of the whole surface generated. 5

3. (*a*) Find the value of x such that the vectors $\{1, 2, 1\}$, $(x, 3, 1\}$ and $(2, x, 0\}$ are linearly dependent. 4

(*b*) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ be a given matrix. Then verify that the matrix A satisfies its own characteristic equation and hence find A^{-1}. 6

(*c*) Find a linear transformation $T : V_3 \to V_4$ whose image is generated by the vectors $\{1, 2, 0, -4\}$ and $\{2, 0, -1, -3\}$. 4

(*d*) Find the eigenvalues and the corresponding eigenvectors of the matrix 6

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$$

4. (*a*) What is the rank of the following matrices : 2 + 2

(*i*) $\begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ and (*ii*) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(*b*) Investigate, for what values of a and b, the system of simultaneous equations

$$x + y + z = 6$$
$$x - 2y + 3z = 10$$
$$x + 2y + az = b$$

has (*i*) no solution, (*ii*) unique solution and (*iii*) an infinite number of solutions. 6

(*c*) (*i*) What is the order of the differential equation whose general solution is
$y(x) = (A + B)\cos(x + C) + D e^x$? 3

(*ii*) Find the differential equation of the family of circles $x^2 + y^2 + 2gx + 2fy + c = 0$. 3

(*d*) Solve the differential equation

$$\left(xy^2 - e^{1/x^3}\right)dx - x^2 y\, dy = 0.$$ 4

Group B

5. (*a*) Eliminate the arbitrary functions f and g from the relation $z(x, t) = f(x + 4t) - g(x - 4t)$ to develop a partial differential equation. 4

(*b*) Solve the differential equation
$$y''(x) - 4y'(x) + 4y(x) = e^{2x}$$
subject to $y'(0) = 1$ and $y(0) = 0$. 6

(*c*) Use Laplace transform to evaluate
$$\int_0^{\infty} \sin x / x \, dx.$$ 4

(*d*) Find the Laplace transform of the function
$$f(t) = \sin t \,;\ 0 < t < \pi$$
$$= 0 \,;\ \pi < t < 2\pi$$
and extended periodically with period 2π. 6

6. (*a*) Prove the following relations between the operators : 3×2

(*i*) $\nabla = 1 - E^{-1}$

(*ii*) $\mu = 1/2\,(E^{1/2} + E^{-1/2})$

(*iii*) $E = hD$

In the above, the symbols have their usual meaning.

(*b*) Calculate the value of
$$\int_0^1 \frac{x}{1+x}\,dx$$
correct to three significant digits taking six intervals by the Trapezoidal rule. 8

(*c*) By constructing a difference table and taking the second order differences as constant, find sixth term of the series 8, 12, 19, 29, 42. 6

7. (*a*) Apply the method of variation of parameters to solve $y''(x) + 4y(x) = \sin 2x$. 8

(*b*) Solve the first order partial differential equation
$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right).$$ 4

(*c*) Consider the following table :

x	20	24	28	32
$f(x)$	0·01427	0·01581	0·01772	0·01996

Using appropriate Newton's formula, compute $f(21)$ and $f(31{\cdot}5)$. 8

8. (*a*) Find the interpolating polynomial for the data :
$f(-1) = 0, f(0) = 1$ and $f(1) = 2$. 6

(*b*) Suppose that during rainy season, on a tropical island, the length of shower has an exponential distribution with an average length of shower 30 sec. What is the probability that a shower will last more than 3 min. If a shower has already lasted for 2 min., what is the probability that it will last for at least one more minute ? 8

(*c*) If X is normally distributed with mean 3 and standard deviation 2, find C such that $P(X > C) = 2P(X \le C)$. Given
$$\int_{-\infty}^{0\cdot 43} \phi(t)\,dt = 1/3.$$ 6

Group C

9. Answer the following : 10×2

(*i*) $\overline{\nabla}\phi$ is a vector normal to the level surface $\phi(x, y, z) =$ constant. Is this statement *true* or *false* ?

(*ii*) If λ is an eigenvalue of a matrix A, then what is the matrix whose eigenvalue is λ^4 ?

(*iii*) No skew-symmetric matrix can be of rank 1. Is the statement *true* or *false* ?

(*iv*) If X is normally distributed with zero mean and unit variance, then what is the expectation of X^2 ?

(*v*) If Fourier transform of a given function $f(x)$ is $F(w)$, then what is the Fourier transform of $f''(x)$?

(*vi*) What is the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$?

(*vii*) Eliminate a and b from the relation $y = a\tan^{-1} x + b$ and develop a differential equation of appropriate order.

(*viii*) Find the complementary function for the differential equation $(D^2 + 4)y = x^2$.

(*ix*) If $L[f(t)] = F(s)$, then what is $L[f(at)]$, $a > 0$.

(*x*) The vectors $(1, 0), (0, 1), (2, 5)$ generate a V_2. Do they form a basis ?

S'15 : 7 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Solve the following system of linear equations by Gauss-elimination method : 6

$$x - 2y + 9z = 8$$
$$3x + y - z = 3$$
$$2x - 8y + z = -5$$

(*b*) Show that 6

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} - \cdots - \frac{1}{n}\right)$$

(*c*) Test the convergence of the series 4

$$\sum_{n=1}^{\alpha} \frac{n!2^n}{n^n}$$

(*d*) Show that $T : R^2 \rightarrow R^3$ defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ is a linear transformation. 4

2. (*a*) Find the extrema of the function $f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$. 7

(*b*) State Green's theorem in vector calculus. Verify the theorem for

$$\oint_C \left[(3x - 8y^2)dx + (4y - 6xy)dy\right]$$

where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$. 7

(*c*) Find the volume of the spindle-shaped solid generated by revolving the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. 6

3. (*a*) If $I_n = \int \frac{\sin n\theta}{\sin\theta} d\theta$, show that $(n - 1)(I_n - I_{n-2}) = 2\sin(n - 1)\theta$. 6

(*b*) Evaluate $\iint \sqrt{4x^2 - y^2}$ over the triangle formed by the straight lines $y = 0, x = 1$ and $y = x$. 8

(*c*) Determine the values of λ and μ so that the equations $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have (*i*) no solution, (*ii*) a unique solution and (*iii*) an infinite number of solutions. 6

4. (*a*) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)u = \frac{3}{x + y + z}$. 5

(*b*) Find the equation of the tangent plane and normal to the surface $z^2 = 4(1 + x^2 + y^2)$ at (2, 2, 6). 5

(*c*) If $\vec{\nabla}\phi = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)\hat{k}$, find the value of ϕ. 5

(*d*) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. 5

Group B

5. (*a*) Apply the method of variation of parameter to solve the equation 5

$$\frac{d^2y}{dx^2} + y = \sec^3 x \tan x$$

(*b*) Prove that

$$f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$$

where Δ, Δ^2 and Δ^3 are forward difference operators of first, second and third orders, respectively. 5

(*c*) If A and B are independent events, then show that A' and B' are independent. 5

(*d*) A car hire firm has two cars which it hires out on daily basis. The number of demand for a car on each day is distributed as a Poisson distribution with average number of demand per day 1·5. Calculate the portion of days on which neither car is used and the portion of days on which some demand is refused. Given : $e^{-1.5} = 0.2231$. 5

6. (*a*) Solve $(x^4 + y^4)dx - xy^3dy = 0$. 5

(*b*) Approximate $\int_0^\pi \frac{\sin x}{x}$ by Simpson's one-third rule with seven ordinates. 5

(*c*) Find the Fourier sine transform of

$$f(x) = \begin{cases} 1, & 0 \le t \le \pi \\ 0, & t > \pi \end{cases}$$ 5

(*d*) Solve the simultaneous equations

$$\frac{dx}{dt}+3x+y=e^t,\ \frac{dy}{dt}-x+y=e^{2t}$$ 5

7. (*a*) The marks obtained by 1000 students in a final examination are found to be approximately normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be between 70 and 75 (both inclusive). Given the area under normal curve, $\phi(z)=1/\sqrt{2\pi}\exp(-z^2/2)$ between $z=0$ and $z=1$ is 0·3413. 5

(*b*) Solve the differential equation by Laplace transform :

$$\frac{d^2y}{dt^2}+9y=1, y(0)=1, y(\pi/2)=-1$$ 5

(*c*) Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=3x+1$ 5

(*d*) Construct the interpolation polynomial for the function $y=\sin\pi x$ choosing the points $x_0=0,\ x_1=1/6,\ x_2=1/2$. 5

8. (*a*) Using the method of separation of variable, solve

$$\frac{\partial^2u}{\partial x^2}=\frac{\partial u}{\partial t},\ u(0,t)=0, u(4,t)=0, u(x,0)=\sin 3x.$$ 8

(*b*) A certain stimulus administered to each of 12 patients resulted in following increase of blood pressures : 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be calculated that stimulus is accompanied by an increase in blood pressure, given that for 11 degrees of freedom the value of $t_{0·5}$ is 2·201 ? 6

(*c*) Solve $\frac{d^2y}{dx^2}+a^2y=\tan ax\,(a\neq 0)$. 6

Group C

9. Answer the following : 10 × 2

(*i*) If $\phi(x, y, z)=0$, find the value of

$$\left(\frac{\partial x}{\partial y}\right)_{z=\text{constant}}\times\left(\frac{\partial y}{\partial z}\right)_{x=\text{constant}}\times\left(\frac{\partial z}{\partial x}\right)_{y=\text{constant}}$$

(*ii*) The reduction formula of $I_n=\int_0^{\pi/2}\sin^n x\,dx$ is $I_n=(n-1/n)\,I_{n-2}$. Find the value of $\int_0^{\pi/2}\cos^4 x\,dx$.

(*iii*) Determine the constant m so that the vector $\vec{v}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+mz)\hat{k}$ is solenoidal.

(*iv*) Find the region of validity of the expansion of $\log(1+5x)$.

(*v*) Evaluate $(\Delta^2/E)x^2$, if the interval of difference is 2.

(*vi*) Find the sum of eigenvalues of the inverse of matrix

$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & -6\\ 0 & 0 & 5\end{bmatrix}$$

(*vii*) Find the value of x for which the three vectors (5, 2, 3), (7, 3, x) and (9, 4, 5) are linearly dependent.

(*viii*) What is the probability that a leap year selected at random will contain 53 wednesday ?

(*ix*) Find the Fourier sine transform of the function $f(x)=1,\ 0<x<l$.

(*x*) Find the particular integral of

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=\sin 3x$$

W'15: 7 FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $y = (\sin^{-1}x)^2$, prove that

$$(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} - n^2 y_n = 0$$ 5

(*b*) Test the convergence of following series for $x > 0$:

$$1 + \frac{1}{2}x + \frac{1}{5}x^2 + \frac{1}{10}x^3 + \ldots + \frac{x^n}{n^2+1} + \ldots + \infty$$ 5

(*c*) If $z = uv$, $u^2 + v^2 - x - y = 0$, $u^2 - v^2 + 3x + y = 0$, find $\partial z / \partial x$. 5

(*d*) Show that the vectors (2, 1, 4), (1, –1, 2) and (3, 1, –2) form a basis of R^3. 5

2. (*a*) Expand $e^{\sin x}$ by Maclaurin's theorem up to the term in x^5. 5

(*b*) If $I_n = \int x^n (a-x)^{1/2} dx$, prove that

$$(2n+3) I_n = 2an I_{n-1} - 2x^n (a-x)^{3/2}$$

Hence, evaluate

$$\int_0^a x^2 \sqrt{(ax - x^2)}\, dx.$$ 7

(*c*) Show that length of the arc of a parabola $y^2 = 4ax$, which is intercepted between the points of intersection of the parabola and the straight line $3y = 8x$, is a $[\log 2 + (15/16)]$. 4

(*d*) If ϕ is a scalar function and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that

$$\text{div}(\vec{r}\,\phi) = 3\phi + \vec{r} \cdot \text{grad}\,\phi.$$ 4

3. (*a*) Let T be the linear operator on R^2 defined by

$$T(x, y) = (4x - 2y, 2x + y)$$

Compute the matrix of T relative to the basis $\{(1, 1), (-1, 0)\}$. 5

(*b*) If $Z = A\, e^{-gx} \sin(nt - gx)$, where A, g, n are positive constants, satisfies the heat conduction equation

$$\frac{\partial u}{\partial t} = C\, \frac{\partial^2 u}{\partial x^2},$$

show that $g = \sqrt{(n/2c)}$. 5

(*c*) Evaluate

$$\iint_A xy\,dx\,dy,$$

where A is the domain bounded by the x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. 5

(*d*) Find the work done in moving a particle from (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1) if the field of force $\vec{F}$ is

$$\vec{F} = (3x^2 + 6y)\,\hat{i} - 14yz\,\hat{j} + 20xz^2\,\hat{k}$$ 5

4. (*a*) Find a matrix P which transforms the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

to the diagonal form. Write the diagonal matrix. 5

(*b*) Divide 24 into three parts such that the continued product of the first, the square of the second and the cube of the third may be a maximum. 5

(*c*) Evaluate the integral

$$\int_0^\infty \int_0^x x e^{-x^2/y}\, dy\, dx$$

by changing the order of integration. 5

(*d*) Use Stoke's theorem to evaluate

$$\iint_S \left(\nabla \times \vec{A}\right) \cdot \hat{n}\, ds$$

where $\vec{A} = y\hat{i} + (x - 2xz)\,\hat{j} - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 5

Group B

5. (*a*) Solve the differential equation

$$\left(1 + x^2\right) \frac{dy}{dx} + 2yx - 4x^2 = 0$$

and obtain the cubic curve satisfying this equation and passing through the origin. 5

(*b*) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = \cos x + x^2$$ 5

(*c*) Find the Laplace transform of

$$f(t) = te^{-2t}\sin 3t$$ 5

(*d*) A packet of 10 items from a factory, whose products are often of poor quality, contains four defective items in it. If three items are drawn without replacement from the packet and X denotes the random variable representing the total number of defective drawn, find the mean and standard deviation of X. 5

6. (*a*) Solve the initial value problem by Laplace transforms method :

$$\frac{d^2x}{dt^2} + x = 3\cos 2t$$

when $x = dx/dt = 0$ at $t = 0$. 5

(*b*) Solve the following equation by the method of variation of parameters :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$$ 5

(*c*) Determine a partial differential equation having the expression

$$Z = f(2x + y) + g(x - y) - xy.$$ 5

(*d*) The average life of a certain type of small motor is 10 years with a standard deviation of two years. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 3% of the motors that fail, how long a guarantee should be offered ? Assume that the life-time of a motor follows a normal distribution. Given that $\phi(-1{\cdot}881) = 0{\cdot}03$, where

$$\phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-x^2/2}dx$$ 5

7. (*a*) Two analysts A and B analyzed a series of ten metals and respectively obtained the results below for the percentage of a particular metal in them.

Alloy No. :	1	2	3	4	5	6	7	8	9	10
Analyst A :	7	9	8	10	8	11	9	8	9	8
Analyst B :	11	7	10	10	9	10	10	9	11	11

Test whether the two analysts differ significantly. (given 5% value of t for d.f. 9 is 2·26.) 5

(*b*) From the following table, estimate $f(3{\cdot}5)$ and $f(6{\cdot}5)$ by using appropriate interpolation formulae : 5

x	3	4	5	6	7
$f(x)$	6	24	60	120	150

(*c*) Find the value of $\log 2^{1/3}$ from

$$\int_0^1 \frac{x^2}{1+x^3}dx$$

using Simpson's one-third rule by dividing the interval into four equal sub-intervals. 5

(*d*) Solve the differential equation by the method of separation of variables :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x,0) = 4e^{-x}.$$ 5

8. (*a*) If $f(t)=\begin{cases} e^{-sx}\, g(t), & t>0 \\ 0, & t<0 \end{cases}$

prove that the Fourier transform at $f(t)$ is equal to the Laplace transform of $g(t)$. 5

(*b*) Use Lagrange's interpolation formula to find a polynomial to the data :

x	-1	0	2	3
y	-8	3	1	12

Hence, estimate y when $x = 1$. 5

(*c*) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = x, \quad 0 \le x \le 50$$
$$= 100 - x, \quad 50 \le x \le 100$$

Find the temperature $u(x, t)$ at any time. 10

Group C

9. Answer the following : 10×2

(*i*) If the function $f(x)$ is continuous in the closed interval $0 \le x \le 1$ and differentiable in the open interval $0 < x < 1$, then $f'(x_1) = f(1) - f(0)$ where $0 < x_1 < 1$.

(*ii*) Expand the function $f(x^2/1 + x)$ by Taylor's theorem.

(*iii*) If $u = \sin^{-1}\left(\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, then prove that

$$\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$$

(*iv*) Show that the basis of the vector space $V = \{x(t);\ x''' - 6x'' + 11x' - 6x = 0\}$ is $\{e^t, e^{2t}, e^{3t}\}$.

(*v*) If the product of two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

is -4, find all eigenvalues of matrix A.

(*vi*) If $f(r)$ is function and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $\vec{r}\cdot\nabla f(r) = r\, f'(r)$, where $r = |\vec{r}|$.

(*vii*) If X is a Poisson variate such that $P(X = 2) = 2P(X = 1)$, find standard deviation of X.

(*viii*) Find the inverse Laplace transform of $\log[(s-1)/s]$.

(*ix*) Obtain the differential equation for the expression

$$Z = (x-a)^2 + (y-b)^2.$$

(*x*) Find the integrating factor of the equation

$$(x^2y^2 + xy + 1)\, y\,dx + (x^2y^2 - xy + 1)\, x\,dy = 0.$$

S'16 : 7 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) A function $f(x)$ is defined in the interval $0 \le x \le 2$ by

$$f(x) = x^2 + x + 1; \quad 0 \le x \le 1$$
$$= 2x + 1 \quad , \quad 1 \le x \le 2$$

Examine the derivability of $f(x)$ at $x = 1$. 5

(*b*) If $y(x) = e^{\cos^{-1} x}$, then obtain a relation involving y_1, y_2 and y. Further use this relation to obtain the result

$$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + 1)\, y_n = 0$$

In the above, y_r $(r = 1, 2, \ldots n \ldots)$ represents the rth derivative of $y(x)$ with respect to x. 5

(*c*) Find the directional derivative of $xy + yz + zx$ in the direction of the vector $i + 2j + 2k$ at the point $(1, 2, 0)$. 5

(*Turn Over*)

(*d*) If S be the length of an arc of $3ay^2 = x(x-a)^2$ measured from the origin to the point (x, y), then show that

$$3S^2 = 4x^2 + 3y^2.$$ 5

2. (*a*) If $x = u - v$ and $y = u^2 - v^2$, then evaluate the Jacobian

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$ 5

(*b*) Obtain the reduction formula for

$$\int (\sin^{-1} x)^{12}\, dx.$$

Hence, evaluate

$$\int_0^1 (\sin^{-1} x)^4\, dx.$$ 8

(*c*) Evaluate

$$\int_S A \cdot n\, dS$$

where $A = (x + y^2)i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ included in the first octant. 7

3. (*a*) Evaluate the definite integral

$$\int_{\sqrt{2}/3}^{\sqrt{3}/2} \frac{dx}{\sqrt{4-9x^2}}.$$ 5

(*b*) Using Gaussian elimination and back substitution, solve the set of following equations :

$$\left.\begin{aligned} x + y + 2z &= 4 \\ 2x + 2y + z - w &= -1 \\ y + z + w &= 6 \\ y - z + 2w &= 5 \end{aligned}\right\}.$$ 8

(*c*) State the Green's theorem in the plane. Verify this theorem for the function

$$A = (2xy - x^2)i + (x^2 + y^2)j$$

over the region enclosed by $y = x^2$ and $x = y^2$. 7

4. (*a*) Find the eigenvalues and eigenvectors of a 3×3 matrix

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & -1 \end{bmatrix}$$ 6

(*b*) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ 6

(*c*) Give the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix},$$

construct a matrix P which transform the matrix A to its diagonal form. Hence, evaluate A^4. 4 + 4

Group B

5. (*a*) Considering an equation of circle

$$(x-a)^2 + (y-b)^2 = c^2$$

where (a, b) are the co-ordinates of the centre and c, the radius, obtain a differential equation of an appropriate order and degree. 6

(*b*) Prove that the whole length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$. 8

(c) State the Rolle's theorem of differential calculus. Verify whether the theorem is applicable to the function

$$f(x) = 2 + (x-1)^{2/3},\ 0 \le x \le 2$$ 6

6. (a) Solve the differential equation

$$(x^2 - y^2)\,dx - xy\,dx = 0.$$ 5

(b) Solve

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y(x) = (1-e^2)^2$$ 5

(c) Verify that the series

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \cdots\infty$$

is convergent. 5

(d) Show that the function

$$f(x, y) = \tan^{-1}(y/x) + \sin^{-1}(x/y)$$

is a homogeneous function of x and y and then compute

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}.$$ 5

7. (a) Given the relation

$$z(x, y) = f(x+y) + g(x, y)$$

by eliminating the arbitrary functions f and g, develop a partial differential equation of appropriate order and degree. 6

(b) Use the well-known Lagrange's theorem to solve the partial differential equation

$$x^2(y-z)\frac{\partial z}{\partial x} + y^2(z-x)\left(\frac{\partial z}{\partial y}\right) = z^2(x-y)$$ 6

(c) (i) Find the Laplace transform of the function $f(t) - (1 - e^t)/t$. 4

(ii) Find the Fourier sine transform of the function $f(t) = e^{-|t|}$. 4

8. (a) Find the interpolating polynomial for the data : $f(-1) = 0$, $f(0) = 1$, $f(1) = 2$. 5

(b) Mr. X has one share in a lottery in which there is one prize and two blanks. Mr. Y has three shares in a lottery in which there are three prizes and six blanks. Compare the probability of X's success to that of Y's success. 5

(c) Solve the following one-dimensional wave equation : 10

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2};\ u = u(x,t);\ 0 \le x \le l$$

$$\left.\begin{array}{l} u(0,t) = 0 \\ u(l,t) = 0 \end{array}\right\}\ \text{Boundary Conditions}$$

$$\left.\begin{array}{l} \left(\dfrac{\partial u}{\partial t}\right)_{t=0} = 0 \\ \text{and } u(x,0) = x(l-x) \end{array}\right\}\ \text{Initial Conditions}$$

Group C

9. Answer the following : 10 × 2

(i) Given the matrix

$$[A] = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix},$$

find the matrix $[B]$ such that the product matrix is

$$[A][B] = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}.$$

(*ii*) What is the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$?

(*iii*) Let $f(x) = 2x^2\hat{i} + 3y^2\hat{j} + 4z^2k$. Find curl div $f(x)$.

(*iv*) Given

$$f(x) = x^2y + y^2x,$$

compute

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}.$$

(*v*) Find the value of the integral

$$\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}\, dx$$

(*vi*) Find the Laplace transform of the function

$$\frac{\partial^2 f(x)}{\partial x^2},$$

given $\mathscr{L}[f(x)] = F(s)$.

(*vii*) State the mean value theorem of differential calculus.

(*viii*) Let $a = 2\hat{i} + 3\hat{j} - k$ and $b = \hat{i} - 3\hat{k}$. Express the vector x in the equation $3a + 2x = b$ in terms of $\hat{i}, \hat{j}, \hat{k}$.

(*ix*) Is the infinite series

$$\frac{1}{4\cdot 7\cdot 10} + \frac{4}{7\cdot 10\cdot 13} + \frac{9}{10\cdot 13\cdot 16} + \cdots\infty$$

convergent or divergent ?

(*x*) Find the first derivative of $f(x)$ at $x = 0{\cdot}3$, where $f(x)$ is given by

x	0·1	0·2	0·3	0·4	0·5	0·6
$f(x)$	0·425	0·475	0·400	0·450	0·525	0·575

W'16: 7FN: AN209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A**, ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a,b,etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2y_2 + xy_1 + y = 0$$

and $x^2y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2+1)y_n = 0$ 5

(*b*) Examine the convergence of the series

$$\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + \ldots\ldots$$ 5

(*c*) If $u = \log r$ where $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r^2}.$$ 5

(*Turn Over*)

(*d*) Show that the set $H = \{(x, y, z) : ax + by + cz = 0, a, b, c \in R^3\}$ is a subspace of R^3. What is its dimension? 5

2. (*a*) By Maclaurin's theorem find the expansion of $y = \sin(e^x - 1)$ upto and including the term in x^4. 5

(*b*) If $I_n = \int \frac{dx}{(x^2+a^2)^n}$, prove that

$$2(n-1)a^2 I_n = \frac{x}{(x^2+a^2)^{n-1}} + (2n-3)I_{n-1}.$$

Hence evaluate $\int_0^1 (x^2+1)^{-3} dx$ 5

(*c*) Find the volume of the solid formed by the revolution of loop of the curve

$$y^2(a+x) = x^2(3a-x)$$

about the x-axis. 5

(*d*) If T be the linear operator on R^2 defined by

$$T(x, y) = (4x - 2y,\ 2x + y).$$

Compute the matrix of T relative to the ordered basis $\{\vec{e}_1, \vec{e}_2\}$ where $\vec{e}_1 = (1, 1)$, $\vec{e}_2 = (-1, 0)$. 5

3. (*a*) If $f(x,y) = \frac{1}{x^3} + \frac{1}{x^2 y} + \frac{1}{x^3 + 5y^3}$, show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$$

Hence, find the value of

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2}.$$ 5

(*b*) Find the directional derivative of the divergence of the vector $\vec{u} = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ at the point (2, 1, 2) in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$. 5

(*c*) Test for consistency and solve the equations : 5

$$3x_1 + 4x_2 + 5x_3 + 6x_4 = 7,$$
$$4x_1 + 5x_2 + 6x_3 + 7x_4 = 8$$
$$5x_1 + 6x_2 + 7x_3 + 8x_4 = 9$$
$$10x_1 + 11x_2 + 12x_3 + 13x_4 = 14$$

(*d*) Evaluate

$$\int_0^a \int_0^x \frac{\cos y \, dy dx}{\sqrt{(a - x(a - y)}}.$$ 5

4. (*a*) Find the minimum value of $x^2 + y^2 + z^2$ given that $ax + by + cz = p$ and justify your result. 5

(*b*) Use the Gram-Schmidt orthonormalization process to determine the orthonormal basis for the given set of linearly independent vectors

$$\vec{x}_1 = (1, 0, 1),\ \vec{x}_2 = (-1, 1, 0)$$
$$\vec{x}_3 = (-3, 2, 0).$$ 5

(*c*) Use divergence theorem to evaluate

$$\iint_S \left(x^3 dydz + x^2 y\, dz dx + x^2 z\, dx dy\right)$$

where S is the surface of the region bounded by the closed cylinder $x^2 + y^2 = a^2$, $z = 0$ and $z = b$. 5

(*d*) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 5

Group B

5. (*a*) Solve:

$$(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$$ 5

(*b*) Use Laplace transforms to evaluate

$$\int_0^\infty t^2 e^{-t} \sin t \, dt$$ 5

(*c*) Given the following table, find the pressure of steam at 142 °C and 175 °C by using appropriate interpolation formulae:

Temp °C :	140	150	160	170	180
Pressure : (kgf/cm^2)	3·685	4·854	6·302	8·076	10·220

5

(*d*) A firm installed as an experimental measure three computers at a central place in a big industrial complex. The number of demands for a computer on each day is distributed as a Poisson variate with mean 2. Calculate the proportion of days on which neither computer is used (11) some of the demands is refused. 5

6. (*a*) Solve:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 11y = x e^{-2x} + e^{-2x} \sin 2x$$ 5

(*b*) The following table gives the values of a function at equal intervals:

x :	0·0	0·5	1·0	1·5	2·0
$f(x)$:	0·3989	0·3521	0·2420	0·1295	0·0540

Evaluate $f'(0), f''(0)$ and $\int_0^2 f(x)\,dx$. 5

(*c*) The average number of miles per litre of brand *A* petrol found by running 5 similar cars under identical

conditions was 6·6 with a standard deviation of 0·12. When brand *B* of petrol was used in 6 similar cars under conditions identical to the former the average mileage came out 6·3 with a standard deviation of 0·13. Could we, on the basis of this data, conclude that brand *A* petrol gives significantly more mileage than brand *B* ?
(Given 5% values of *t* for d.f. 9, 10 and 11 are 2·26, 2·23 and 2·20 respectively.) 5

(*d*) Solve:

$$x\left(z^2 - y^2\right)\frac{\partial z}{\partial x} + y\left(x^2 - z^2\right)\frac{\partial z}{\partial y} = z\left(y^2 - x^2\right)$$ 5

7. (*a*) Apply the method of variation of parameter to solve

$$\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$$ 5

(*b*) The following data relates the production (x) and import (y) of an item in India during five consecutive years:

Production (x) : (in 1000 tons)	48·2	47·1	41·9	42·2	43·7
Imports (y) :	1·6	1·8	2·6	2·7	2·8

(*i*) Compute the correlation coefficient between production and import.

(*ii*) Estimate the best value of y for $x = 45·5$. 5

(*c*) The mode of a certain frequency curve $y = f(x)$ is very near to $x = 9$ and the values of the frequency density $f(x)$ for $x = 8·9, 9, 9·3$ are respectively 0·30, 0·35 and 0·25. Calculate the approximate value of the mode by using Lagrange's interpolation formula. 5

(*d*) In an explosive factory cordite sticks are manufactured by two processes were as below :

	Defective	Non-defective
Process A :	100	900
Process B :	60	440

Does this data indicate that the Process A is superior to the process B ? (5% values of chi-square for d.f. 1, 2 and 3 are respectively 3·811, 5·99, 7·81) 5

8. (*a*) The equations of motion of a particle are given by

$$\frac{dx}{dt} + my = 0,$$

$$\frac{dy}{dt} - mx = 0$$

show that the path of the particle is a circle. 5

(*b*) The mean inside diameter of a sample of 200 washers produced by a machine is 5·02 mm and the standard deviation is 0·05 mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4·96 to 5·08 mm, otherwise the washers are considered defective. Determine the number of defective washers in the sample produced by the machine, assuming the diameters are normally distributed. (Given ϕ (1·2) = 0·8944 where

$$\phi\,(z) = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-\frac{11}{2}x^2}\,dx\;)$$ 5

(*c*) A string of length 1 is initially at rest in equilibrium position and each of its point is given the velocity

$$v_0 \sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{2\pi x}{l}\right)$$

where $0 < x < l$, at $t = 0$. Determine the displacement function $y\,(x, t)$. 10

Group C

9. Answer the following : 10 x 2

(*i*) A function $f(x)$ is defined by

$$f(x) = x\sin\frac{1}{x}\text{ in } (-1, 1)$$

$$= 0 \text{ when } x = 0$$

Are the conditions of first mean value theorem of differential calculus satisfied in this case ?

(*ii*) If $x^2 + y^2 + z^2 - 2xyz = 1$, show that

$$\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}} = 0$$

(*iii*) Let λ be an eigenvalue of a square matrix A with corresponding eigenvector X. Let $P(\lambda) = a_0 + a_1\lambda + a_2\lambda_2 + \cdots + a_n\lambda$. Define $P(A) = a_0 I + a_1 A + a_2 A^2 + \cdots + a_n A^n$. Show that $p\,(A)\,X = p\,(\lambda)\,X$.

(*iv*) If ϕ satisfies Laplace equation, show that $\nabla\phi$ is both solenoidal and irrotational.

(*v*) If P and D are respectively modal and spectral matrices of a square matrix A, then show that

$$A^n = PD^n\,p^{-1}$$

(*vi*) If $E^2ux = x^2$ and $h = 1$, find ux.

(*vii*) Is $\frac{1}{3}\sin 6x$ the particular integral of the equation

$$\frac{d^2y}{dx^2} + 36y = 4\cos 6x\;?$$

(*viii*) If $F(s)$ is the complex Fourier transform of $f(x)$, prove that

$$F\{f(x)\cos 9x\} = \frac{1}{2}\{F(s+a) + F(s-9)\}$$

(*ix*) Show that the vectors $\vec{u} = (1+i, 2i)$ and $\vec{v} = (1, 1+i)$ are linearly independent over the real field.

(*x*) The diameter X of an electric cable is a random variate with probability density function

$$f(x) = kx(1-x),\ 0 \le x \le 1$$

Show that $E(X) = \frac{1}{2}$.