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BTECH NOTES SERIES

Strength of Materials (Mechanics of Solids)

(As Per AICTE/Technical Universities Syllabus)

CONTENT

(Use bookmarks to navigate)

BENDING & SHEAR STRESSES IN BEAMS AND CURVED BEAMS

BENDING STRESSES IN BEAMS

The Equations Expressing relationship between bending moment(M) acting at any section in a beam and the bending stress (σ) at any point in this same section.

$$\frac{\sigma}{y} = \frac{E}{R} \qquad (1)$$

$$\frac{M}{I} = \frac{\sigma}{y} \qquad (2)$$

(In some books, "f" is used in place of "σ")

Example

Derive the bending equation i.e.; M/I = σ/y = E/R

Solution

With reference to Fig. (a), let us consider any two normal sections AB and CD of a beam at a small distance δL apart (i.e., AC = BD = δL). Let AB and CD intersect the neutral layer at M and N respectively.

Let;

M = bending moment acting on the beam

θ = Angle subtended at the centre by the arc.

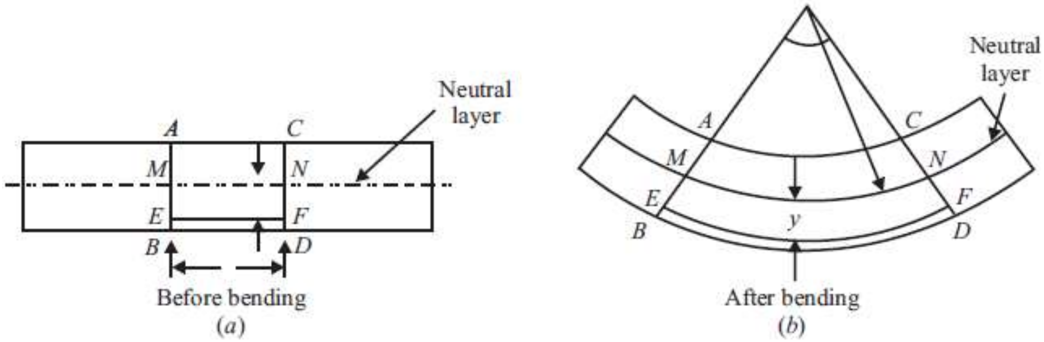
R = Radius of curvature of the neutral layer M′N′.

At any distance 'y' from the neutral layer MN, let us consider a layer EF.

Fig. (b) shows the beam due to sagging bending moment. After bending, A′B′, C′D′, M′N′ and E′F′ represent the final positions of AB, CD, MN and EF respectively.

When produced, A′B′ and C′D′ intersect each other at O subtending an angle θ radian at O, which is the centre of curvature.

Since δL is very small, arcs A′C′, M′N′, E′F′ and B′D′ may be taken as circular.



Now, strain in the layer EF due to bending is given by e = (E′F′ – EF)/EF = (E′F′ – MN)/MN

Since MN is the neutral layer, MN = M′N′

$$e = \frac{E'F' - M'N'}{M'N'} = \frac{(R+\theta) - R\theta}{R\theta} = \frac{y\theta}{R\theta} = \frac{y}{R} \qquad (1)$$

Let; σ = stress set up in the layer EF due to bending and E = Young's modulus of the material of the beam.

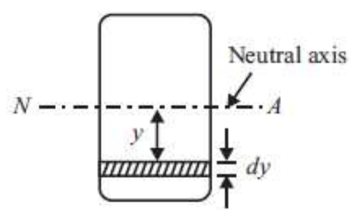
Then $$E = \frac{\sigma}{e} \Rightarrow e = \frac{\sigma}{E} \qquad (2)$$

From equation (1) and (2)

$$\frac{y}{R} = \frac{\sigma}{E}$$

or $$\frac{\sigma}{y} = \frac{E}{R} \qquad (3)$$

With reference to following figure.



At the distance 'y', let us consider an elementary strip of very small thickness dy. We have already assumed that 'σ' is the bending stress in this strip.

Let dA = area of this elementary strip.

Then, force developed in this strip = σ.dA.

Then, elementary moment of resistance due to this elementary force is given by dM = σ.dA.y

Total moment of resistance due to all such elementary forces is given by

$$\int dM = \int \sigma . dA . y$$

or $$M = \int \sigma . dA . y \quad (4)$$

From eq (3)

$$\sigma = y \, x \frac{E}{R}$$

Putting this value of σ in Eq. (4), we get

$$M = \int y \, x \frac{E}{R} \, x \, dA \, x \, y = \frac{E}{R} \int dA . y^2$$

But $$\int dA . y^2 = 1$$

where I = Moment of inertia of the whole area about the neutral axis NA.

M = (E/R) . I

M/I = E/R

Thus; M/I = σ/y = E/R

Where;

M = Bending moment

I = Moment of Inertia about the axis of bending i.e. I_{xx}

y = Distance of the layer at which bending stress is consider

(We take always the maximum value of y, i.e., distance of extreme fibre from N.A.)

E = Modulus of elasticity of the beam material.

R = Radius of curvature

ASSUMPTIONS IN BENDING EQUATION

- Normal sections of the beam, which were plane before bending, remain plane after bending.
- The material is homogeneous and isotropic, so that it has the same elastic properties in all directions.
- The beam is initially straight and of uniform cross section.
- Modules of elasticity in tension and compression are equal.
- It obeys Hook’s law i.e. the stress is proportional to strain within the elastic limit.
- The radius of curvature of the bam before bending is very large in comparison to its transverse dimensions.
- The resultant pull or push across transverse section is zero.

SECTION MODULUS

$$M = \sigma \frac{I}{y} = \sigma Z$$

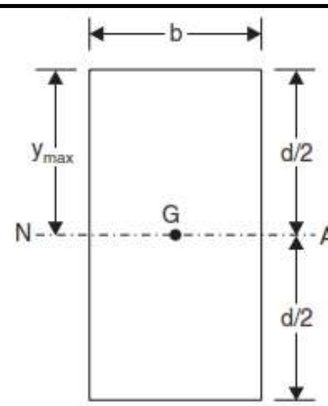
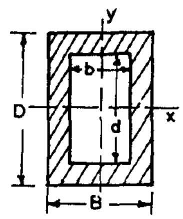
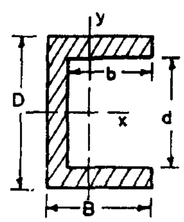
Z = I/y is called section modulus. The section modulus represents strength of section. Greater the value of Z, stronger will be the section.

For Circular Shaft (Z) = I/y = $(\pi D^4/64)/D/2 = (\pi D^3/32)$

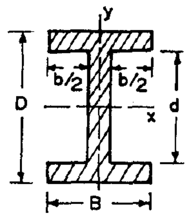
For Hollow Shaft (Z) = I/y = $\{\pi(D^4 - d^4)/64\}/D/2 = (\pi/32)(D^4 - d^4)/D$

For Rectangular section (Z) = I/y = $(bd^3/12)/d/2 = bd^2/6$

MOMENT OF INERTIA/SECTION MODULUS OF VARIOUS SECTIONS

Section	Figure	Formula
Rectangle		$I_{xx} = \frac{1}{12}bd^3$ $I_{yy} = \frac{1}{12}db^3$ $I_{AB} = \frac{1}{3}bd^3$ $Z_{xx} = \frac{I_{xx}}{y_{max}} = \frac{1}{6}bd^2$
Hollow Rectangular Section		$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right)$ $Z_{xx} = \frac{BD^3 - bd^3}{6D}$
Channel Section		$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right)$

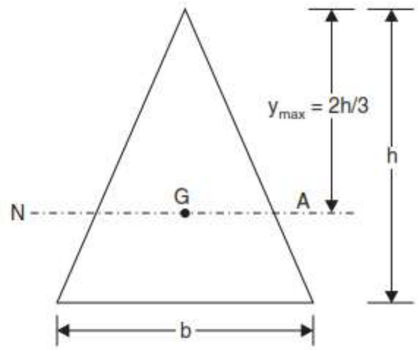
I Section



$$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right)$$

where b is B - thickness of web and d is (D - 2 x thickness of flange)

Triangle

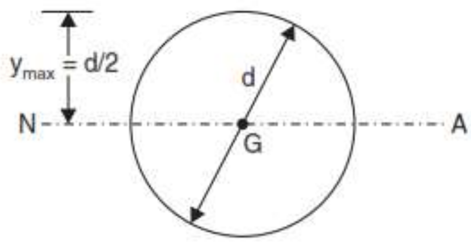


$$I_{xx} = \frac{1}{36}bh^3$$

$$I_{AB} = \frac{1}{12}bh^3$$

$$Z = \frac{bh^2}{24}$$

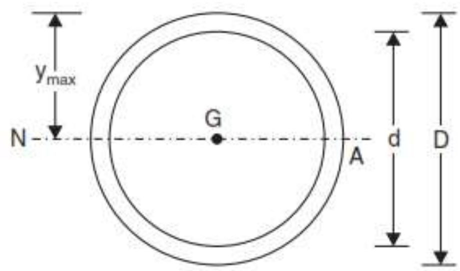
Circle of diameter d



$$I_{xx} = \frac{\pi}{64}d^4$$

$$Z_{xx} = \frac{\pi}{32}d^3$$

Hollow circular section (ring) having inside diameter (d) and outside diameter (D)



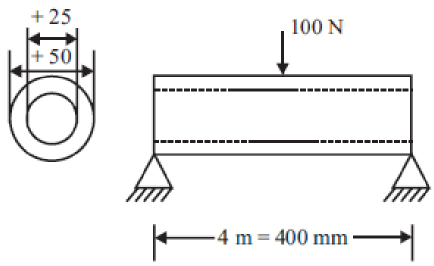
$$I_{xx} = \frac{\pi}{64}(D^4 - d^4)$$

$$Z_{xx} = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$$

Example

A beam made of C.I. having a section of 50 mm external diameter and 25 mm internal diameter is supported at two points 4 m apart. The beam carries a concentrated load of 100N at its centre. Find the maximum bending stress induced in the beam.

Solution



This problem is the case of simply supported beam with load at its mid-point, and in this case maximum bending moment

$$= M = WL/4 = (100 \times 4)/4 = 100\text{ Nm} = 100 \times 10^3\text{ N.mm} \quad (1)$$

Let I = Moment of inertia = $\frac{\pi}{64}(d_0{}^4 - d_i{}^4) = \frac{\pi}{64}(50^4 - 25^4) = 287621.4\,mm^4$

$$y = d/2 = 50/2 = 25\text{mm} \quad (2)$$

We know that

Maximum stress during Bending moment (at y = 25mm) = M.y/I

$$= (100 \times 10^3 \times 25)/287621.4 = 8.69\text{ N/mm}^2$$

Maximum bending stress = 8.69 N/mm^2

Example (UTU 2022, 5 marks)

A rectangular beam 300 mm deep is simply supported over a span of 4 m. Determine the uniformly distributed load per metre which the beam may carry, if the bending stress should not exceed 120 N/mm^2. Take I =8 × 10^6 mm^4.

Solution

Depth, d = 300 mm

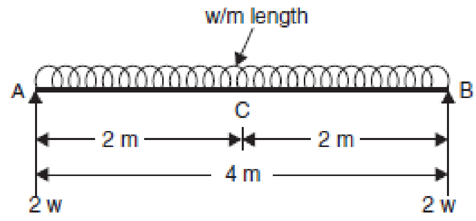
Span, L = 4 m

Max. bending stress, = 120 N/mm^2

Moment of inertia, I = 8 x 10^6 mm^4

Let, w = U.D.L. per metre length over the beam in N/m.

The bending stress will be maximum, where bending moment is maximum. For a simply supported beam carrying U.D.L., the bending moment is maximum at the centre of the beam (i.e., at point C of given figure.)



∴ Max BM = 2w x 2 - 2w x 1 = 2w Nm = 2000w N-mm

Now $M = \sigma_{\max} Z$

where $Z = I / y_{\max} = 8x10^6 / 150$

∴ $$2000w = 120x\frac{8x10^6}{150}$$

Solving, w = 3200 N/m

Example

A steel bar 10 cm wide and 8 mm thick is subjected to bending moment. The radius of neutral surface is 100 cm. Determine maximum and minimum bending stress in the beam.

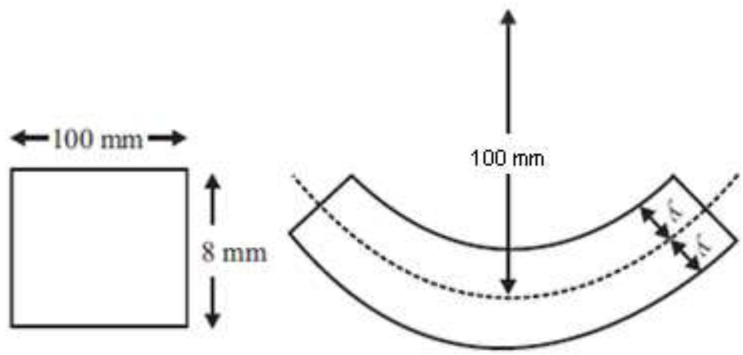
Solution

Assume for steel bar $E = 2 \times 10^5$ N/mm^2

$y_{max} = 4$ mm

$R = 1000$ mm

$\sigma_{max} = E.y_{max}/R = (2 \times 10^5 \times 4)/1000 = 800$ N/mm^2



We get maximum bending moment at lower most fibre, Because for a simply supported beam tensile stress (+ve value) is at lower most fibre, while compressive stress is at top most fibre (–ve value).

$\sigma_{max} = 800$ N/mm^2

$y_{min} = -4$ mm

R = 1000 mm

$\sum\sigma_{min} = E.y_{min}/R = 2 \times 10^5 \text{ x } (-4)/1000$

$\sigma_{min} = -800$ N/mm^2

Example

A steel wire, 6 mm diameter, is bent to a circular shape with a mean radius of 3m. Calculate the maximum stresses induced in the wire if the Young's modulus of elasticity is 210 GNm^{-2}.

Solution

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\therefore \quad \sigma = \frac{E}{R} y = \frac{210 \text{ x } 10^3}{3 \text{ x } 10^3} x \frac{1}{2} x6 = 210 \text{ N/mm}^2$$

(y = distance between N.A. and outer fibre where max stress occurs)

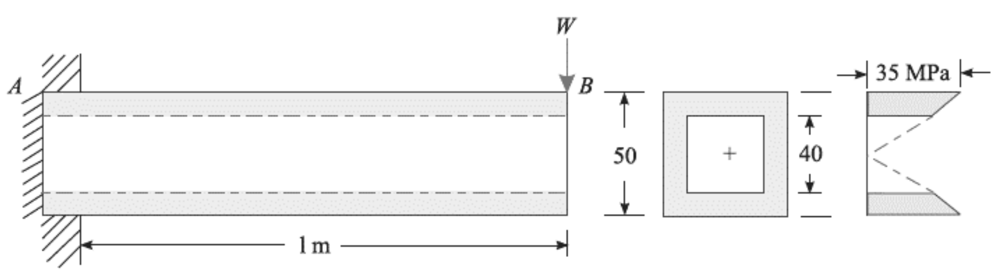
Problem

A steel flat 10 cm wide and 2 cm thick is bent into a circular arc of 50 metres radius. Find the maximum intensity of stress induced. Take E = 2.05 x 10^5 N/mm^2.

Answer: 41 N/mm^2

Example (UTU 2022, 10 marks)

A hollow square section with outer and inner dimensions of 50 mm and 40 mm respectively is used as a cantilever of span 1 m depicted in figure. How much concentrated load can be applied at the free end of the cantilever, if the maximum bending stress is not to exceed 35 MPa?



Solution

Let W = Concentrated load that be applied at the free end of the cantilever.

We know that moment of inertia of the hollow square section,

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{BB^3}{12} - \frac{bb^3}{12} = \frac{B^4}{12} - \frac{b^4}{12}$$

$$= \frac{50^4}{12} - \frac{40^4}{12} = 307.5x10^3\ mm^4$$

∴ Modulus of section

$$Z = \frac{I}{y} = \frac{307.5x10^3}{25} = 12300\,mm^3$$

and maximum bending moment at the fixed end of the cantilever subjected to a point load at the free end, M = Wl = W(l x 10^3) = 1 x 10^3 W

Maximum bending stress (σ_{max}),

$$35 = \frac{M}{Z} = \frac{1x10^3 W}{12300}$$

∴ W = 430.5 N

Example

A timber joist of 6 metres span has to carry a load of 15 kN/metre. Find the dimensions of the joist if the maximum permissible stress is limited to 8 kN/mm^2. The depth of the joist has to be twice the width.

Solution

Given joist (beam) is carrying a udl of 15 kN/m.

Hence $$M = \frac{wl^2}{8} = \frac{15x36}{8} = 67.5\,kN-m = 67.5x10^6\ N-mm$$

We know $$M = \sigma Z$$

∴ $$Z = \frac{67.5x10^6}{8} = 8.44x10^6\ mm^3$$

But $$Z = \frac{1}{6}bd^2 = \frac{1}{6}b(2b)^2 = \frac{2}{3}b^3$$

∴ $$b^3 = \frac{8.44x10^6 x3}{2} = 12.66x10^6$$

Solving b = 233 mm and d = 466 mm

Example

A 30 cm x 16 cm rolled steel joist(beam) of I section has flanges 11 mm thick and web 8 mm thick. Find the safe uniformly distributed load that this section will carry over a span of 5 metres if the permissible skin stress is limited to 120 N/mm^2.

Solution

$$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right) = \frac{1}{12}(16x30^3 - 15.2x27.8^3) = 8800\,cm^4 = 88x10^6\ mm^4$$

$$Z_{xx} = \frac{88x10^6}{150} = 58.6x10^4\ mm^3$$

Let w = safe uniformly distributed load in kN/m.

$$\therefore \qquad M_{max} = \frac{wl^2}{8} = \frac{wx25}{8}kNm = \frac{wx25}{8}x10^6\ N-mm = 3.125x10^6\ N-mm$$

$$\therefore \qquad \sigma_{max} = \frac{M}{Z} = \frac{3.125x10^6 w}{58.4x10^4} = \frac{312.5}{58.6}w\ N/mm^2$$

But this is equal to 120 N/mm^2.

$$\therefore \qquad \frac{312.5}{58.6}w = 120$$

Solving w = 22.5 kN/m

Problem

A beam of symmetrical section 30 cm deep and I = 12000 cm^4 carries udl of 16 kN/m. Calculate the maximum span of the beam if the maximum bending stress is not to exceed 160 N/mm^2. With this span, calculate the maximum central load if the bending stress is not to exceed the limit given above.

Answer: 6.93 m, 55.4 kN

Problem

A timber joist of 4 metre span has to carry a total load of 80 kN uniformly distributed over the length. Calculate the dimensions of the joist if the maximum permissible stress is limited to 8 N/mm^2. The ratio of depth to width is to be 1.5.

Answer: 24 cm x 36 cm

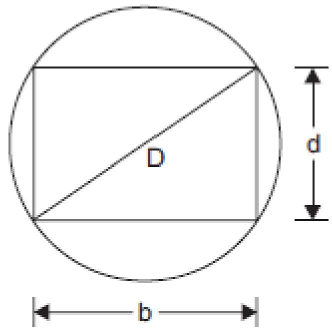
Example

A single beam of rectangular cross section is to be cut from a log of wood of diameter D. What must be the ratio of the depth to the breadth(d/b) for maximum bending strength ?

Solution

diameter = D

Total depth d = $\sqrt{D^2 - b^2}$



Now $M = \frac{\sigma}{y}.I$

$= \sigma.\frac{(1/12).b.(\sqrt{D^2 - b^2})^3}{(1/2)\sqrt{D^2 - b^2}}$

$= (1/6)\sigma.b.(D^2 - b^2)$

As $\frac{dM}{db} = 0$

∴ $D^2 - 3b^2 = 0$

∴ $D = \sqrt{3}\, b$

∴ $b = D/\sqrt{3}$

∴ $d = D\sqrt{\frac{2}{3}}$

∴ $d/b = \sqrt{2}$

Example

Find the dimension of the strongest rectangular beam that can be cut out of a log of 25 mm diameter.

Solution

$b^2 + d^2 = 25^2$

$d^2 = 25^2 - b^2$

Since $M/I = \sigma/y$; $M = \sigma(I/y) = \sigma.Z$

M will be maximum when Z will be maximum

$Z = I/y = (bd^3/12)/(d/2) = bd^2/6 = b.(25^2 - b^2)/6$

The value of Z maximum at dZ/db = 0;

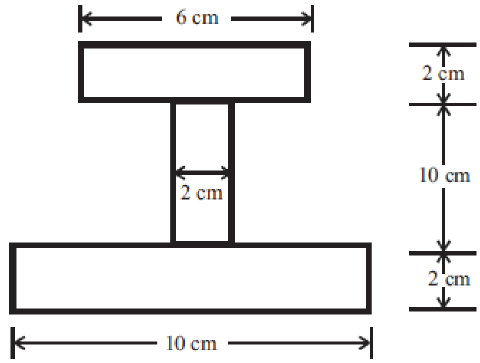
i.e.; $d/db[25^2 b/6 - b^3/6] = 0$

$25^2/6 - 3b^2/6 = 0$

b = 14.43 mm; d = 20.41 mm

Example

A beam having I – section is shown in fig is subjected to a bending moment of 500 Nm at its Neutral axis. Find maximum stress induced in the beam.



Solution

Since diagram is symmetrical about y–axis.

$Y = (A_1y_1 + A_2y_2 + A_3y_3)/(A_1 + A_2 + A_3)$

$A_1 = 6 \times 2 = 12\ cm^2$

$A_2 = 10 \times 2 = 20\ cm^2$

$A_3 = 10 \times 2 = 20\ cm^2$

$y_1 = 2 + 10 + 1 = 13$ cm

$y_2 = 2 + 5 = 7$ cm

$y_1 = 1 = 1$ cm

putting all the values; we get

$Y = \{12 \times 13 + 20 \times 7 + 20 \times 1\}/(12 + 20 + 20)$

$Y = 6.08$ (1)

Moment of inertia about an axis passing through its C.G. and parallel to X – X axis.

$I = I_{XX1} + I_{XX2} + I_{XX3}$

$I_{XX1} = I_{G1} + A_1h_1^2 = bd^3/12 + A_1(Y - y_1)^2$

$I_{XX2} = I_{G2} + A_2h_2^2 = bd^3/12 + A_2(Y - y_2)^2$

$I_{XX3} = I_{G3} + A_3h_3^2 = bd^3/12 + A_3(Y - y_3)^2$

$I = [bd^3/12 + A_1(Y – y_1)^2]+ [bd^3/12 + A_2(Y – y_2)^2] + [bd^3/12 + A_3(Y – y_3)^2]$

$= [(6 \times 2^3)/12 + 12 \times (6.08 – 13)^2] + [(2 \times 10^3)/12 + 20 \times (6.08 – 7)] + [(10 \times 2^3)/12 + 20 \times (6.08 – 1)^2]$

$I = 1285\ cm^4$

Distance of C.G. from upper extreme fibre

$y_c = 14 – 6.08 = 7.92cm$

Distance of C.G. from lower extreme fibre

$y_t = 6.08$ cm

Therefore we will take higher value of y i.e.;

$y = 7.92$,

which gives the maximum value of stress but compressive in nature.

At $y_t = 6.08$ cm, we get the tensile stress, but at this value of y, we don't get the maximum value of stress, Since Our aim is to find out the maximum value of stress in the beam which we get either on top most fibre or on bottom most fibre, depending upon the distance of fibre from centre of gravity. We always take the maximum value of y, because;

$M/I = \sigma_{max}/y;\ \sigma_{max} = M.y/I,$

$\sigma_{max} = 500 \times 10^2 \times 7.92/1285 = 308.2\ N/cm^2$

$\sigma_{max} = 308.2\ N/cm^2$

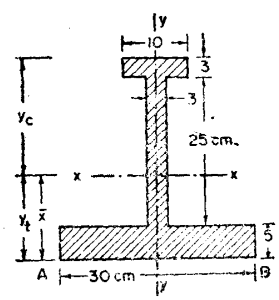
Example

The compression flange of the cast iron girder is 10 cm wide and 3 cm deep, the tension flange 30 cm wide and 5 cm deep and the web 25 cm x 3 cm. Find (a) moment of inertia about N.A. (b) the load per metre run which may be carried over a 3 m span by a beam simply supported at its ends if the maximum permissible stresses are 95 N/mm² in compression and 25 N/mm² in tension.

Solution

(a) Total area = (10 x 3) + (25 x 3) + (30 x 5) = 255 cm²

$$\bar{x} = \frac{(30x41.5)+(75x17.5)+(150x2.5)}{255} = 10.3\,cm$$



$$I_{AB} = \left(\frac{1}{3}x27x5^3\right)+\left(\frac{1}{3}x3x30^3\right)$$

$$+\left\{\left(\frac{1}{12}x10x3^3+\left(30x31.5^2\right)\right)\right\} = 57850\,cm^4$$

$\therefore \quad I_{xx} = 57850 - 255x10.3^2 = 30850\,cm^4 = 308.5x10^6\,mm^4$

(b) Now y_c = 33 - 10.3 = 22.7 cm = 227 mm

y_t = 10.3 cm = 103 mm

$$\frac{f_c}{y_c} = \frac{f_t}{y_t}$$

i.e. $f_c = \frac{f_t}{y_t} y_c = f_t \frac{227}{103} = 2.2f_t$

When f_t reaches 25 N/mm^2, f_c = 25 x 2.2 = 55 N/mm^2, which is lower than the permissible limit. Should f_c be allowed to go to 95 N/mm^2, f_t will evidently be more than 25 N/mm^2, which is not permissible. Hence allowable tensile stress will be the criterion for the determination of strength.

$\therefore \quad Z_t = \frac{I_{xx}}{y_t} = \frac{308.5x10^6}{103} mm^3$

$\therefore \quad M_t = f_t Z_t = 25x\frac{308.5x10^6}{103} = 74.8x10^6\,N-mm$

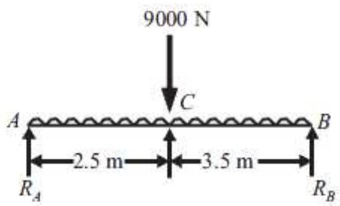
$B.M. = \frac{wl^2}{8} = \frac{wx9}{8}$, w is in kN/m.

$\therefore \quad 1.125wx10^6 = 74.8x10^6$

$\therefore$ w = 66.4 kN/m

Example

A beam is freely supported on supports as shown in fig carries a UDL of 12 KN/m and a concentrated load. If the stress in beam is not to exceed 8N/mm^2. Design a suitable section making the depth twice the width.



For finding out Maximum bending moment which is at point ‘C’

$\sum M_A = 0;$

$-R_B \times 6 + 9000 \times 2.5 + 12000 \times 6 \times 3 = 0$

Since $R_A + R_B = 9000 + 12000 \times 6 = 81000$ N

$R_B = 39750$ N; $R_A = 41250$ N

For shear force equation;

Consider right hand side of the section; we get

$SF_{1-1} = 39750 – 12000 . X = 0; X = 3.31$ m

Maximum B.M. at X = 3.31m = $R_B . X – 12000 . X^2/2$

Maximum B.M. = 65836×10^3 N–mm

$\sigma = y.M/I$ or; $M = \sigma.I/y = \sigma[(bd^3/12)/d/2] = \sigma.bd^2/6$

$65836 \times 10^3 = [8 \times d/2 \times d^2]/6$

d = 462 mm, b = 231 mm

Example (UTU 2023, 10 marks)

A timber beam of rectangular section is to support a load of 20 kN over a span of 4 m. If the depth of the section is to be twice the breadth, and the stress in the timber is not to exceed 60 N/mm, find the dimensions of the cross-section. How would you modify the cross-section of the beam if it were a concentrated load placed at the centre with the same ratio of breadth to depth.

Solution

Given that d = 2b; $y_{max} = d/2 = b$

$\therefore Z = bd^2/6 = b(2b)^2/6 = (2/3)b^3$

(a) When the load of 20 kN is uniformly distributed over a span of 4 m

w = 20/4 = 5 kN/m

$$\therefore \quad M_{\max} = \frac{wL^2}{8} = \frac{5x4^2}{8} = 10\,kNm = 10x10^6\,N-mm \qquad (1)$$

$$\text{Now} \quad M_r = fZ = 60x\frac{2b^3}{3} = 40b^3\,N-mm \qquad (2)$$

Equating the moment of resistance to the maximum BM, we have

$40b^3 = 10 \text{ x } 10^6$

From this, b = 63 mm

Hence, d = 2b = 126 mm

(b) When the load of 20 kN is placed at the mid span,

$$M_{\max} = \frac{WL}{4} = \frac{20x4}{4} = 20\,kNm = 20x10^6\,N-mm$$

$M_r = 40b^3$ N-mm as calculated earlier.

∴ $40b^3 = 20 \times 10^6$

Giving b = 80 mm

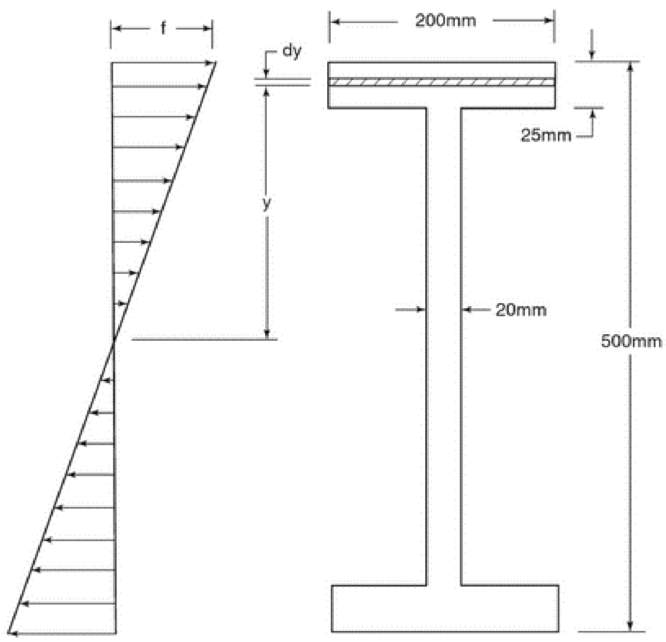
Hence, d = 2b = 160 mm

Example (JNTUK 2021, 8 marks)

An I section beam has 200 mm wide flanges and an overall depth 500 mm. Each flange is 25 mm thick and the web is 20 mm thick. At a certain section bending moment is M. Find what percentage of M is resisted by flanges and web.

Solution

See following diagram.



Due to symmetry centroid and hence neutral axis is at middle of the depth of I-section

Consider an element of thickness dy at distance y from Neutral Axis. Let f be maximum bending stress. Since bending stress varies linearly with depth, stress on the element.

$$= \frac{y}{250} f$$

Area of the element = 200dy

∴ Force on the element

$$= \frac{y}{250} f(200dy)$$

Its moment of resistance = moment of resisting force about NA

$$= \frac{y}{250} f(200dy)y$$

∴ Moment of resistance of top flange

$$= \int_{225}^{250} \frac{200}{250} y^2 fdy$$

Due to symmetry, moment of resistance of lower flange is also same as that of top.

Moment of resistance of flanges

$$= 2\int_{225}^{250} \frac{200}{250} y^2 fdy$$

$$= 1.6f\left[\frac{y^3}{3}\right]_{225}^{250} = 2258333.3f$$

Now, moment of inertia of I section is

$$I = \frac{1}{12}\left[200x500^3 - 180x450^3\right]$$

$$= 7.16458 \text{ x } 10^8 \text{ mm}^4$$

∴ Moment of entire section

$$M = f\frac{I}{y_{\max}} = f\frac{7.16458x10^8}{250} = 2865833.3f$$

∴ Percentage moment resisted by flanges

$$= \frac{2258333.3}{2865833.3}x100 = 78.802\%$$

∴ Percent moment resisted by web

= 100 – 78.802 = 21.198

Example (AKTU 2019, 10 marks)

A water main of 500 mm internal diameter and 20 mm thick is running full. The water main is of cast iron and is supported at two points 10 m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3 respectively.

Solution

Given Data:

Internal dia., D_i = 500 mm = 0.5 m

Thickness of pipe, t = 20 mm

∴ Outer dia., $D_0 = D_i + 2 \times t = 500 + 2 \times 20 = 540$ mm = 0.54 m

Weight density of cast iron = 72000 N/m^3

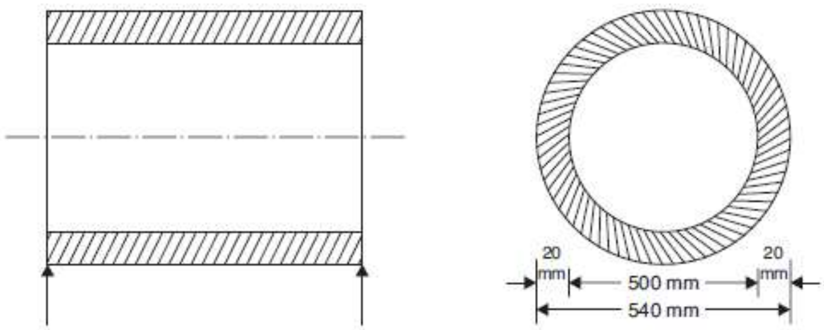
Weight density of water = 10000 N/m^3

Internal area of pipe = $\frac{\pi}{4} D_i^2$ = 0.1960 m^2

This is also equal to the area of water section.

∴ Area of water section = 0.196 m^2

Outer area of pipe = $\frac{\pi}{4} D_0^2 = \frac{\pi}{4}(0.54)^2 m^2$



(a) (b)

Area of pipe section = $\frac{\pi}{4} D_0^2 - \frac{\pi}{4} D_i^2 = \frac{\pi}{4}(0.54^2 - 0.5^2) = 0.0327 m^2$

Moment of inertia of the pipe section about neutral axis

$$I = \frac{\pi}{64}[D_0^4 - D_i^4] = \frac{\pi}{64}[540^4 - 500^4] = 1.105 x 10^9 mm^4$$

Let us now find the weight of pipe and weight of water for one metre length.

Weight of the pipe for one metre run

= Weight density of cast iron × Volume of pipe

= 72000 × [Area of pipe section × Length]

= 72000 × 0.0327 × 1 (∵ Length = 1 m)

= 2354 N

Weight of the water for one metre run

= Weight density of water × Volume of water

= 10000 × (Area of water section × Length)

= 10000 × 0.196 × 1 = 1960 N

∴ Total weight on the pipe for one metre run

= 2354 + 1960 = 4314 N

Hence the above weight is the U.D.L. (uniformly distributed load) on the pipe. The maximum bending moment due to U.D.L. is $w \times L^2/8$, where w = Rate of U.D.L. = 4314 N per metre run.

∴ Maximum bending moment due to U.D.L.,

$$M = \frac{wL^2}{8} = \frac{4314x10^2}{8} = 53925Nm = 53925x10^3\ N - mm$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

∴ $\sigma = \frac{M}{I} y$

The stress will be maximum when y is maximum. But maximum value of

$$y = \frac{D_0}{2} = \frac{540}{2} = 270mm$$

∴ y_{max} = 270 mm

∴ Maximum stress

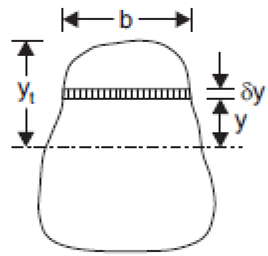
$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{53925x10^3}{1.105x10^9} x270 = 13.18\, N/mm^2$$

SHEAR STRESSES IN BEAMS

DISTRIBUTION OF SHEARING STRESSES

For any section, general equation to compute shear stress(q) at any point distance y from N.A. is given by :

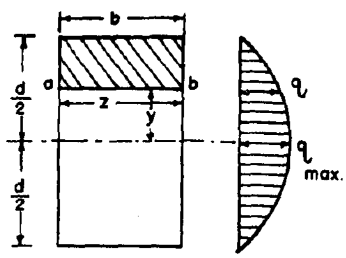
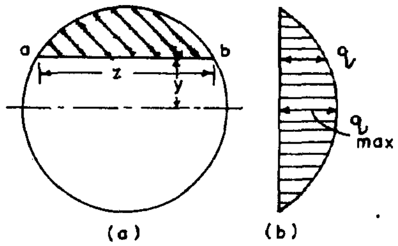
$$q = \frac{F}{bI} A.\bar{y}$$

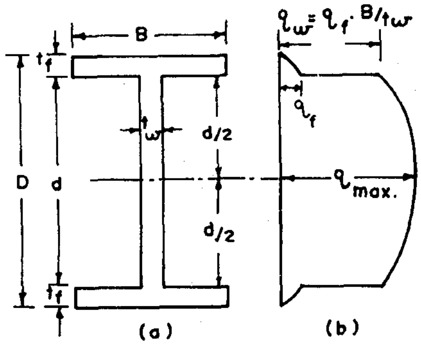


where F = shear force at the section; I = MOI of the section about N.A.; b = width of section at the point where q is required; $A\bar{y}$ = Moment about N.A., A is the area above point where q is required, $\bar{y}$ is distance between centroid of area A and N.A.

Note : *Shear stress is always zero at top and bottom fibre. Bending stress is always zero at N.A.*

SHEAR STRESS DISTRIBUTION OF VARIOUS SECTIONS

Section	Figure	Formula
Rectangular		$q = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$ At y = 0, q = $q_{max} = \frac{Fd^2}{8I}$ at neutral axis (N.A.) $q_{mean} = \frac{F}{bd}$ $q_{max}/q_{mean} = 1.5$
Circular		$q = \frac{F}{3I}(r^2 - y^2)$ At y = 0, $q = q_{max} = \frac{F}{3I}r^2$ at neutral axis(N.A.) $q_{max} = (4/3)q_{mean}$

I Section	 Flange width = B; Flange thickness = t_1; web depth = d = D - $2t_1$; web thickness = t_w	*In the flange* $q = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$ *In the flange, at the junction with web (y = d/2)* $q = \frac{F}{8I}\left(D^2 - d^2\right)$ *In the web* $\frac{F}{8I}\frac{B}{t_w}(D^2 - d^2) + \frac{F}{8I}(d^2 - 4y^2)$ *In the web, at the junction (y = d/2)* $q = \frac{F}{8I}(D^2 - d^2)\frac{B}{t_w}$

Example

Find out equations to represent distribution of shearing stresses in

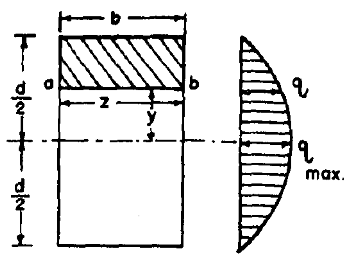
(i) A rectangular section
(ii) A solid circular section

at a distance y above N.A.

Solution

(i) $q = \frac{F}{bI}A\bar{y}$

$$= \frac{F}{b.(bd^3/12)}\left\{b\left\{\frac{d}{2} - y\right\}\right\}\left\{y + \frac{\left\{\frac{d}{2} - y\right\}}{2}\right\}.$$



$$= \frac{6F}{bd^3}\left\{\frac{d^2}{4} - y^2\right\}$$

It shows q varies parabolically with y.

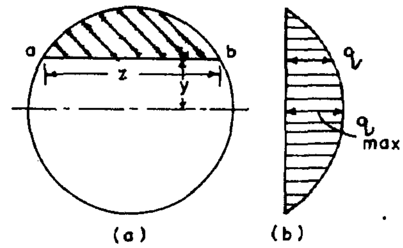
when y = 0, q_{max} = 1.5F/bd

area of shear stress diagram will show total shear force on the section.

(i.e. $\frac{2}{3}$ q_{max} b.d = $\frac{2}{3}$ $\frac{1.5F}{bd}$ = F)

(ii) $$q = \frac{F}{bI}A.\bar{y}$$

$$b = 2\sqrt{r^2 - y^2}$$ [radius is r]



$$I = \frac{\pi}{64}D^4$$

$A\bar{y}$ = moment of area of shaded portion

$$= \int_y^r 2.dy.y = \frac{2}{3}(r^2 - y^2)^{3/2}$$

$$\therefore \quad q = \frac{F}{2\sqrt{r^2 - y^2}\;.(\pi/64)D^4}.\frac{2}{3}(r^2 - y^2)^{3/2} = \frac{F}{3\pi D/64}(r^2 - y^2).$$

$$\therefore \quad q = \frac{64F}{3\pi D^4}(r^2 - y^2)$$

∴ q varies parabolically with y.

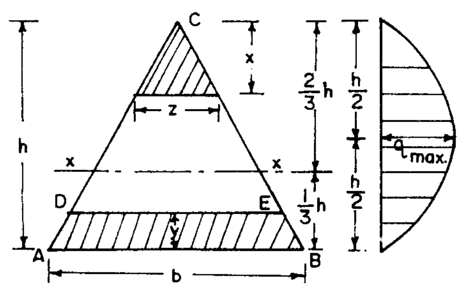
for q_{max} put y = 0.

Example

A beam has for its cross section a triangle with base 'b' and height 'h'. Prove that the maximum intensity of shear stress in any normal section is 3F/bh and that it occurs h/2 from apex.

Solution

$$I = \frac{bh^3}{36}.$$



$$B = \frac{xb}{h},$$

$$A = \frac{1}{2}\left\{\frac{xb}{h}\right\}$$

$$x = \frac{1}{2}\frac{b}{h}x^2$$

$$\bar{y} = \frac{2h}{3} - \frac{2x}{3}$$

$$q = \frac{F}{BI} A\bar{y} = \frac{F}{\frac{xb}{h} \cdot \frac{bh^3}{36}} \cdot \frac{1}{2} \frac{b}{h} x^2 \left\{ \frac{2h}{3} - \frac{2x}{3} \right\}$$

$\therefore$ $$b = \frac{12F}{bh^3}(xh - x^2)$$

But for maximum intensity of shear stress

$$\frac{dq}{dx} = 0$$

$\therefore$ h - 2x = 0,

$\therefore$ x = h/2 for max. shear stress.

$\therefore$ $$q_{\max} = \frac{12F}{bh^3}\left\{ \frac{h}{2}.h \text{ - } \frac{h^2}{4} \right\} = \frac{3F}{bh}$$

Example (JNTUK 2021, 15 marks)

An I section beam 350 mm x 150 mm has a web thickness of 10 mm and a flange thickness of 20 mm. If the shear force acting on the section is 40 kN, find the maximum shear stress developed in the section and also sketch the variation of shear stress across the depth of the section.

Solution

Given

Overall depth, D = 350 mm

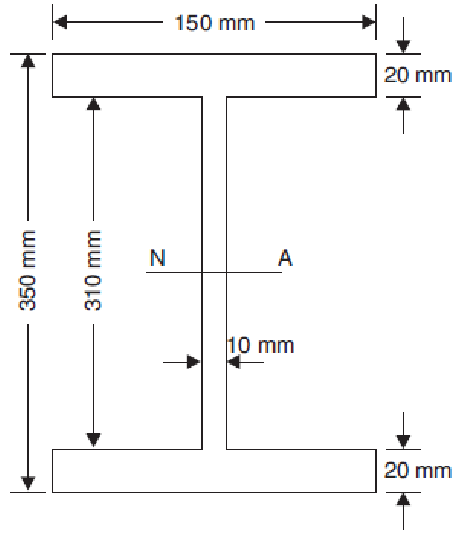
Overall width, B = 150 mm

Web thickness, b = 10 mm

Flange thickness, = 20 mm

$\therefore$ Depth of web, d = 350 – (2 × 20) = 310 mm

Shear force on the section, F = 40 kN = 40,000 N



The maximum shear stress developed in the I-section will be at the neutral axis. This shear stress is given by

$$\tau_{max} = \frac{Fa\bar{y}}{Ib}$$

where F = 40,000 N

$A\bar{y}$ = Moment of the area above the neutral axis about the neutral axis

= Area of flange × Distance of C.G. of the area of flange from neutral axis + Area of web above neutral axis × Distance of the C.G. of this area from neutral axis

$$= (150x20)\left(\frac{310}{2}+\frac{20}{2}\right)+\left(\frac{310}{2}x10\right)\left(\frac{310}{2}x\frac{1}{2}\right)$$

= 615125 mm^3

Moment of inertia of the section about neutral axis,

$$I = \frac{150x350^3}{12} - \frac{140x310^3}{12} = 188375833.4\,mm^4$$

b = 10 mm

$$\therefore \qquad \tau_{max} = \frac{40,000x615125}{188375833.4x10} = 13.06\,N/mm^2$$

Example (PTU 2019, 10 marks)

A beam of I section 50 cm deep and 19 cm wide has flanges 2.5 cm thick and web 1.5 cm thick. It carries a shearing force of 400 kN at a section. Calculate the maximum intensity of shear *stress in the section assuming the moment of inertia to be 64500 cm^4. Also calculate the total shear force carried by the web and sketch the shear stress distribution across the section.*

Solution

Shear stress in flange at the junction with web

$$= \frac{400x10^3 x19x2.5x23.75x10^{-6}}{64500x10^{-8}x0.19} = 3.68MPa$$

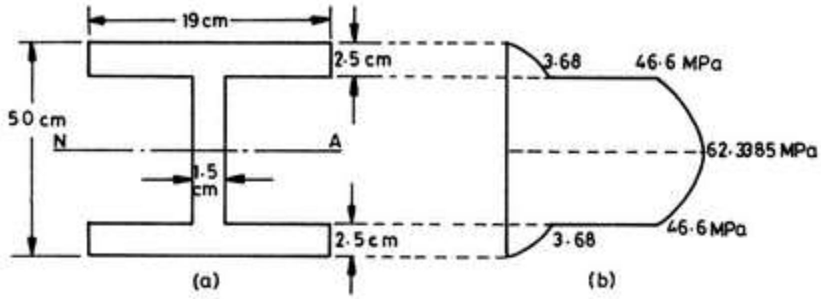
Shear stress in web at the junction with flange

$$= \frac{400x10^3 x19x2.5x23.75x10^{-6}}{64500x10^{-8}x0.015} = 46.6MPa$$

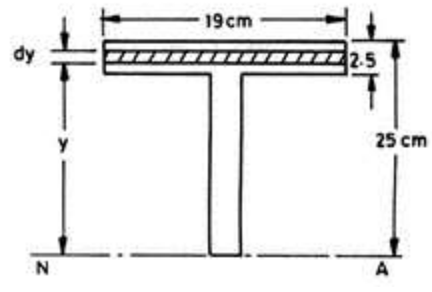
Maximum shear stress at the neutral axis

$$= \frac{400x10^3[19x2.5x23.75+1.5x22.5x11.25]x10^{-6}}{64500x10^{-8}x0.005} = 62.3385MPa$$

The shear stress distribution is shown below.



Now, consider a flange strip of thickness dy at a distance y from the neutral axis.



Shear stress $f_s = \frac{400x10^3\left[19(25-y)\left(\frac{25+y}{2}\right)x10^{-6}\right]}{64500x10^{-8}x0.19}$

$$= \frac{625-y^2}{3225x10^{-8}}$$

Shear force carried by the small strip dy

$$= \frac{625 - y^2}{3225x10^{-8}} x19dyx10^{-4}$$

Shear force carried by one flange

$$= \frac{19}{3225x10^{-4}} \int_{22.5}^{25} (625 - y^2) dy$$

$$= \frac{19}{3225x10^{-4}} \left[625 - \frac{y^3}{3} \right]_{22.5}^{25}$$

= 8898.58 N

Shear force carried by the two flanges

= 2 x 8898.58 = 17797.16 N

Shear force carried by the web

= 400 x 10^3 - 17797.16 = 382202.84 N

Problem

An I-section has the following dimensions

flanges : 150 mm × 20 mm

web : 30 mm × 10 mm

The maximum shear stress developed in the beam is 16.8 N/mm^2. Find the shear force to which the beam is subjected.

Answer: 50 kN

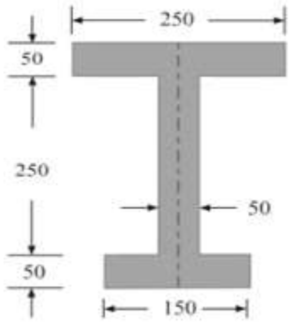
Problem

A 12 cm by 5 cm I-section is subjected to a shearing force of 10 kN. Calculate the shear stress at the neutral axis and at the top of the web. What percentage of shearing force is carried by the web ? Given I = 220 × 10^4 mm^4, area = 9.4 × 10^2 mm^2, web thickness = 3.5 mm and flange thickness = 5.5 mm.

Answer: 27.2 N/mm^2 ; 20.1 N/mm^2 ; 9.5 kN. i.e., 95% of the total

Example (UTU 2022, 10 marks)

A cast-iron bracket subjected to bending, has a cross-section of I-shape with unequal flanges as shown in figure.



If the compressive stress in top flange is not to exceed 17.5 MPa, what is the bending moment, the section can take? If the section is subjected to a shear force of 100 kN, draw the shear stress distribution over the depth of the section.

Solution

$$\bar{y} = \frac{(250x50)325 + (250x50)175 + (150x50)25}{(250x50) + (250x50) + (150x50)} = 198\,mm$$

Distance of centre of gravity from the upper extreme fibre

y_c = 350 - 198 = 152 mm

$$I = \left[\frac{250x50^3}{12} + (250x50)(325-198)^2\right] + \left[\frac{50x250^3}{12} + (50x250)(198-175)^2\right]$$

$$+ \left[\frac{150x50^3}{12} + (150x50)(198-25)^2\right]$$

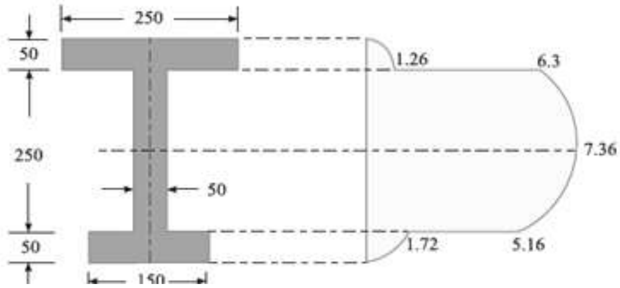
$$= 502 \text{ x } 10^6 \text{ mm}^4$$

Bending moment the section can make

$$= \frac{\sigma_c}{y_c I} = \frac{17.5}{152} x502x10^6 = 57.8x10^6\ N-mm = 57.8\,kNm$$

Shear stress distribution diagram

We know that the shear stress at the extreme edges of both the flanges is zero. Now let us find out the shear stress at the junction of the upper flange and web by considering the area of the upper flange. We know that area of the upper flange,



A = 250 x 50 = 12500 mm

$\bar{y}$ = 152 - (50/2) = 127 mm

B = 250 mm

Shear stress at the junction of the upper flange and web

$$\tau = F\frac{A\bar{y}}{IB} = 100x10^3.\frac{12500x127}{(502x10^6)250} = 1.26\,N/mm^2 = 1.26\,MPa$$

The shear stress at the junction suddenly increases from 1.26 MPa to 1.26 x (250/50) = 6.3 MPa.

Now let us find out the shear stress at the junction of the lower flange and web by considering the area of the lower flange. We know that area of the lower flange.

A = 150 x 50 = 7500 mm^2

$\bar{y}$ = 198 - (50/2) = 173 mm

B = 150 mm

Shear stress at the junction of the lower flange and web

$$\tau = F\frac{A\bar{y}}{IB} = 100x10^3 x\frac{7500x173}{(502x10^6)x150} = 1.72N/mm^2 = 1.72\,MPa$$

Now let us find out the shear stress at the neutral axis, where the shear stress is maximum. Considering the area of the I-section above neutral axis, we know that

$$A\bar{y} = \left[(250x50)x127\right] + \left[(102x50)x\frac{102}{2}\right]mm^3 = 1.848x10^6\,mm^3$$

and b = 50 mm.

Maximum shear stress

$$\tau_{\max} = F\frac{A\bar{y}}{IB} = 100x10^3.\frac{1.848x10^6}{(502x10^6)x50} = 7.36\,N/mm^2 = 7.36\,MPa$$

Shear stress distribution is shown in the figure.

CURVED BEAMS

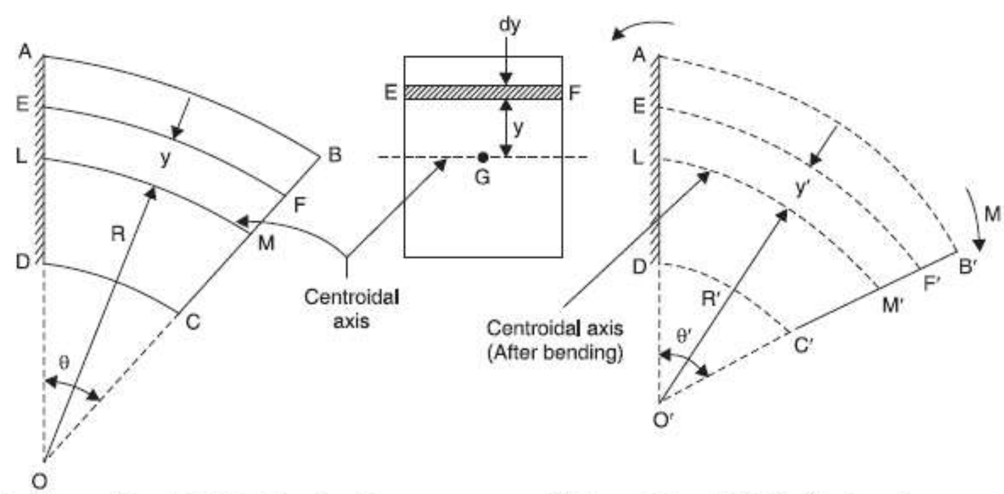
For a straight beam, the bending equation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ is used.

This equation can be applied, with sufficient accuracy, to the beams or bars having small initial curvature. However the machine members such as crane hooks, chain links and rings etc. which are having large initial curvature, the simple bending equation cannot be used.

Generally, if the radius of curvature is more than 5 times the depth of the beam, the beam is said to be having small initial curvature. But if radius of curvature is less than 5 times the depth, the beam is said to be having large initial curvature. Hence for large initial curvature, the radius of curvature is small. Also for curved beams, the neutral and centroidal axes do not coincide.

EXPRESSION FOR STRESSES IN A CURVED BAR

Following figure shows the two positions of a curved bar, one position [i.e., Fig. (a)] is before bending whereas the second position [i.e., Fig. (b)] is after bending when some moment M is applied at the ends of the bar. The centroidal axis is shown by line LM and LM′ in the two positions respectively.



(a) Curved bar ABCD before bending (b) Curved bar AB′C′D after bending

Consider any fibre EF at a distance ‘y’ from the centroidal axis LM. This fibre takes the position of EF′ when the moment M is applied. Then the fibre EF′ will be at a distance of ‘y′’ from the centroidal axis LM′.

Tensile stress will be

$$\sigma = \frac{M}{RA}\left[1+\frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

Compressive stress will be

$$\sigma = \frac{M}{RA}\left[1-\frac{R^2}{h^2}\left(\frac{y}{R-y}\right)\right]$$

where R = radius of curvature of centroidal axis LM

y = distance of fibre EF from centroidal axis LM

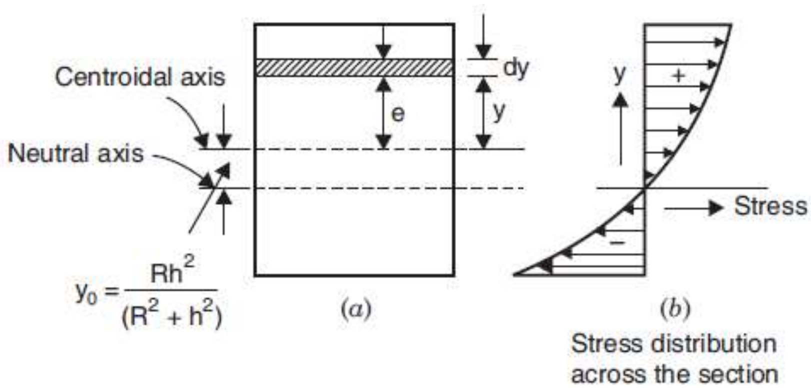
M = uniform bending moment applied to the bar

M = uniform bending moment applied to the bar,

These expressions are known as **Winkler-Bach Formula**.

POSITION OF NEUTRAL AXIS

See following figure.



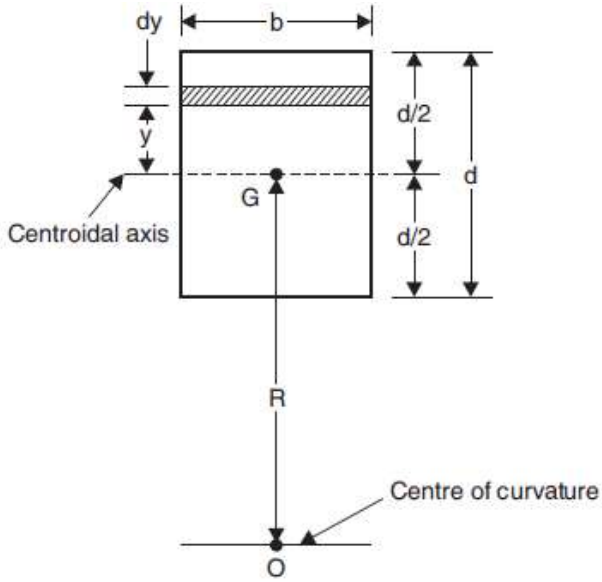
$$y_0 = \frac{Rh^2}{R^2+h^2}$$

Value of h^2 for various sections

$$h^2 = \frac{R^3}{A}\int\left(\frac{1}{R+y}\right)dA - R^2$$

By substituting the values of ‘y’ and dA for various section in this equation, the value of h^2 is obtained

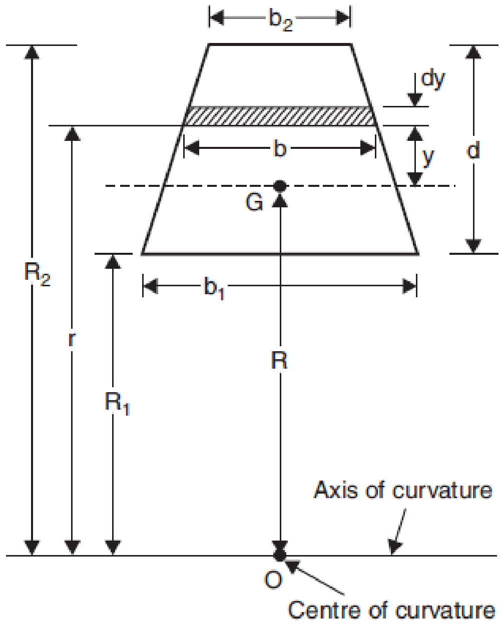
Value of h^2 for rectangular section



$$h^2 = R^2\left[\frac{1}{3}\left(\frac{d}{2R}\right)^2 + \frac{1}{5}\left(\frac{d}{2R}\right)^4 + ...\right]$$

Value of h^2 of trapezoidal section

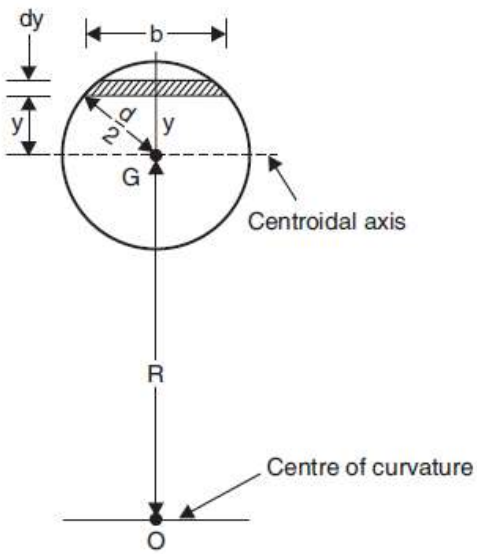
See following figure.



$$h^2 = \frac{R^3}{A}\left[\left\{b_2 + \frac{(b_1 - b_2)}{d}R_2\right\}\log_e\left(\frac{R_2}{R_1}\right) - (b_1 - b_2)\right] - R^2$$

Value of h^2 of a circular section

See following figure.



$$h^2 = \frac{d^2}{16} + \frac{1}{128} \cdot \frac{d^4}{R^2} + ...$$

Example

Determine : (i) location of neutral axis, and (ii) maximum and minimum stress, when a curved beam of rectangular cross-section of width 20 mm and of depth 40 mm is subjected to pure bending of moment + 600 Nm. The beam is curved in a plane parallel to depth. The mean radius of curvature is 50 mm.

Solution

Given data

Curved beam of rectangular section,

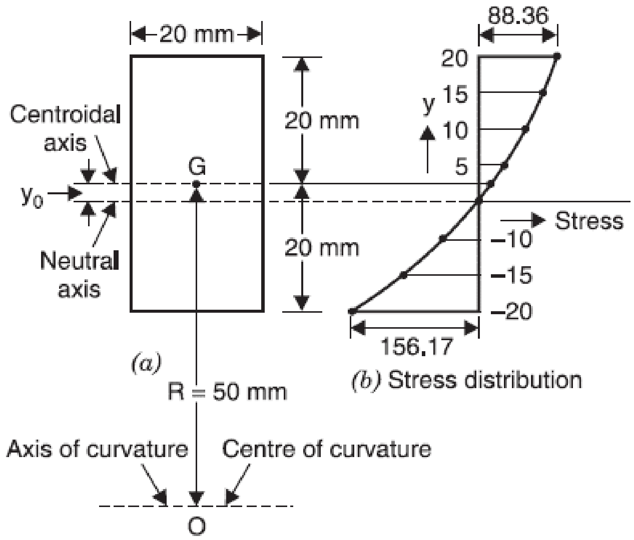
Width, b = 20 mm, depth, d = 40 mm,

∴ Area, A = b × d = 20 × 40 = 800 mm^2

Radius of curvature, R = 50 mm.

Bending moment, M = + 600 Nm = 600 × 1000 = 600,000 Nmm

Figure



Location of neutral axis

$$h^2 = R^2\left[\frac{1}{3}\left(\frac{d}{2R}\right)^2 + \frac{1}{5}\left(\frac{d}{2R}\right)^4\right]$$

$$= 50^2\left[\frac{1}{3}\left(\frac{40}{2x50}\right)^2 + \frac{1}{5}\left(\frac{40}{2x50}\right)^4\right]$$

$$= 146.05$$

Now Let y_0 = distance of neutral axis from centroidal axis

$$y_0 = -\frac{Rh^2}{R^2 + h^2} = -\frac{50x146.05}{50^2 + 146.05} = -2.76\,mm$$

Maximum and minimum stresses

We know $\sigma = \frac{M}{RA}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$

For a given value of M, R and A, the stress will be maximum when y is –ve and maximum. Hence maximum stress will occur at the extreme bottom layer where y = – 20 mm

$$\sigma_{max} = \frac{600,000}{50x800}\left[1 + \frac{50^2}{146.05}\left(\frac{-20}{50-20}\right)\right] = -156.17\,N/mm^2$$

and minimum stress occurs at the extreme top layer where y = 20 mm

$$\sigma_{min} = \frac{600,000}{50x800}\left[1 + \frac{50^2}{146.05}\left(\frac{20}{50+20}\right)\right] = 88.36\,N/mm^2$$

Problem

A sharply curved beam of rectangular section is 10 mm thick and 50 mm deep. If the radius of curvature, R = 60 mm, compute the stress in terms of the bending moment M at a point 20 mm from the outer surface.

Answer: 2.16 kN m, +4.50 N/mm^2

Example (AKTU 2018, 10 marks)

Determine : (i) Location of neutral axis, (ii) Maximum and minimum stresses. When a curved beam of trapezoidal section of bottom width 30 mm, top width 20 mm and height 40 mm is subjected to pure bending moment of + 600 Nm. The bottom width is towards the centre of curvature. The radius of curvature is 50 mm and beam is curved in a plane parallel to depth.

Solution

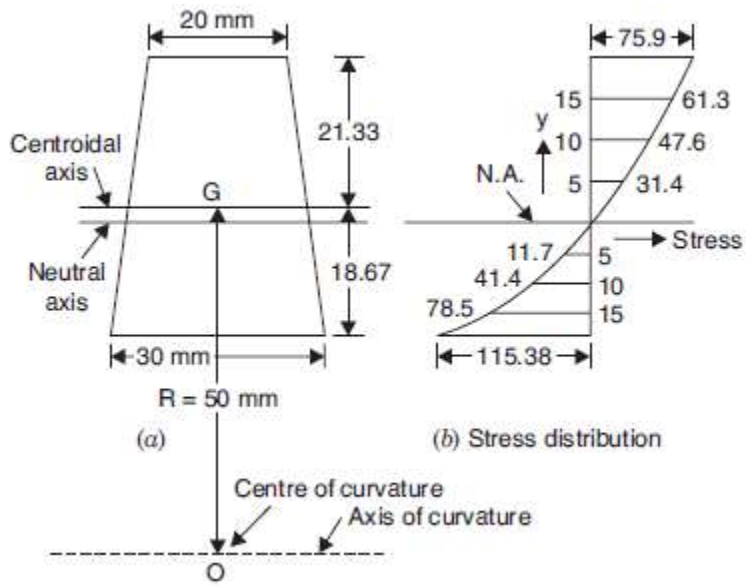
Given data

b_1 = 30 mm, b_2 = 20 mm, d = 40 mm

M = + 600 Nm = 600 × 1000 Nmm

R = 50 mm

Figure



Location of centroidal axis

The distance 'd_1' of centroidal axis from the side b_1 is given by,

$$d_1 = \left(\frac{b_1 + 2b_2}{b_1 + b_2}\right) x \frac{d}{3} = \left(\frac{30 + 2x20}{30 + 20}\right) x \frac{40}{3} = 18.67\, mm$$

$$d_2 = 40 - 18.67 = 21.33 mm$$

R_1, R_2 and area A

$R_1 = R - d_1 = 50 - 18.67 = 31.33$ mm

$R_2 = R + d_2 = 50 + 21.33 = 71.33$ mm

A = Area of trapezoidal section

$$= \left(\frac{b_1 + b_2}{2}\right) xd = \left(\frac{30+20}{2}\right) x40 = 1000 mm^2$$

h^2 for trapezoidal section

$$h^2 = \frac{R^3}{A}\left[\left\{b_2 + \frac{(b_1 - b_2)}{d} R_2\right\}\log_e\left(\frac{R_2}{R_1}\right) - (b_1 - b_2)\right] - R^2$$

$$= \frac{50^3}{1000}\left[\left\{20 + \frac{30-20}{40} x71.33\right\}\log_e\left(\frac{71.33}{31.33}\right) - (30-20)\right] - 50^2$$

$$= 140.34$$

Location of neutral axis

Let y_0 = distance of neutral axis from centroidal axis

$$y_0 = \frac{Rh^2}{R^2 + h^2} = -\frac{50x140.34}{2640.34} = -2.65 mm$$

Maximum and minimum stresses

$$\sigma = \frac{M}{RA}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

The stress will be maximum at the extreme bottom layer where y = – 18.67 mm

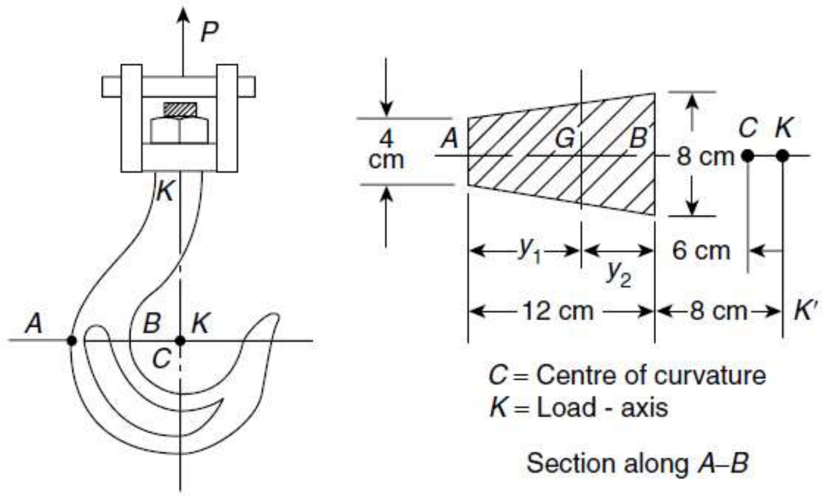
$$\sigma_{max} = \frac{600,000}{50x1000}\left[1 + \frac{50^2}{140.34}\left(\frac{-18.67}{50-18.67}\right)\right] = 115.38 N/mm^2$$

Minimum stress occurs at the extreme top layer where y = 21.33 mm

$$\sigma_{max} = \frac{600,000}{50x1000}\left[1 + \frac{50^2}{140.34}\left(\frac{21.33}{50+21.33}\right)\right] = 75.9 N/mm^2$$

Problem

Determine the maximum compressive and tensile stresses in the critical section of a crane hook lifting a load of 40 kN. The dimensions of the hook are shown in following figure. The line of *application of the load is at a distance of 8 cm from the inner fibre (rounding off the corners of the cross section are not taken into account).*



Answer: 25.2MPa, 57.17MPa

ASSIGNMENT

Q.1. (UTU 2014, 2017, 10 marks): Write about the theory and assumptions in simple bending.

Q.2. (JNTUK 2023, 4 marks): What is bending stress.

Q.3. (PTU 2019, UTU 2022, 2 marks): Define bending and shear stress.

Answer: Bending stress arises when external forces or moments cause an object to bend or deform. This stress can lead to both compressive and tensile forces within the material. This type of stress typically arises in structures or components like beams, bridges, and columns that are subjected to loads that cause them to bend.

Shear stress occurs when the material or fluid's adjacent layers move relative to each other due to an applied force. This results in a component of the force per unit area acting parallel to the surface. This force attempts to deform the material by causing one portion of it to slide past another. It is typically represented by the symbol "τ or σ".

Q.4. (AKTU 2017, 2 marks): Explain: (i) Section Modulus (ii) Modular ratio

Answer: Section Modulus is a geometric property used to calculate the bending stresses in a structural member. It is defined as the ratio of the moment of inertia of a section about its centroidal axis and to the distance of the extreme layer from the neutral axis.

The modular ratio is the ratio of the modulus of elasticity of two different materials. If E_1 and E_2 are Young's modulus of two materials, then the ratio E_1/E_2 or E_2/E_1 is known as modular ratio.

Q.5. (JNTUK 2023, 4 marks): Find the section modulus of a triangular section

Q.6. (AKTU 2020, 2 marks): Define section modulus in case of a beam subjected to bending.

Q.7. (AKTU 2018, PTU 2020, 2 marks): What do you mean by "simple/pure bending"? What are the assumptions made in the theory of simple bending?

Q.8. (GTU 2020, JNTUK 2019, 4 marks): Explain assumptions made in theory of pure bending.

Q.9. (JNTUK 2019, 7.5 marks): Derive the bending equation for a beam subjected to simple bending.

Q.10. (PTU 2019, UTU 2022, 10 marks): Stating the assumptions made, derive the complete flexural formula

$$\frac{M}{I_{NA}} = \frac{\sigma}{y} = \frac{E}{R}$$

Q.11. (AKTU 2018, 2 marks): Show that for a beam subjected to pure bending, neutral axis coincides with the centroid of the cross-section.

Q.12. (JNTUK 2021, 7 marks): Derive the flexure formula.

Q.13. (AKTU 2018, 10 marks): Derive the expression for shearing stress at any section on a beam, also show the distribution of shearing stress over a rectangular section.

Q.14. (AKTU 2019, PTU 2020, 2 marks): Draw Shear stress distribution for rectangular section.

Q.15. (AKTU 2020, GTU 2021, 10 marks): Prove that the maximum shear stress is 3/2 times of the average shear stress in beam of rectangular section subjected to a shear force.

Q.16. (JNTUK 2019, 15 marks): A beam of square section of side a is used with a diagonal in the vertical position. If the vertical shearing force at the cross-section is S, show that the shear stress at the neutral axis is equal to the mean shear stress. Also find where the shear stress is maximum. Find also the ratio of the maximum shear stress to the mean shear stress.

Q.17. (AKTU 2018, 2 marks): Show that for a beam subjected to pure bending, neutral axis coincides with the centroid of the cross-section.

Answer: The neutral axis is an axis in the cross-section of a beam (a member resisting bending) or shaft along which there are no longitudinal stresses or strains. If the section is symmetric, isotropic and is not curved before a bend occurs, then the neutral axis is at the geometric centroid. All fibres on one side of the neutral axis are in a state of tension, while those on the opposite side are in compression.

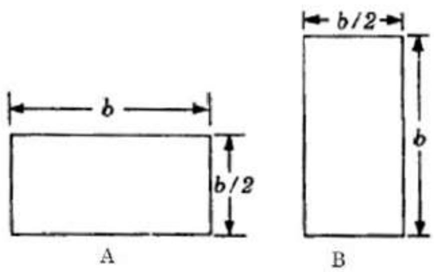
Q.18. (GTU 2020, 3 marks): Draw shear stress distribution diagram for : (1) I section (2) Circular section (3) Triangular section

Q.19. (JNTUK 2023, 4 marks): Draw Shear stress distribution for circular section.

Q.20. (UTU 2022, 5 marks): A rectangular beam 300 mm deep is simply supported over a span of 4 m. Determine the uniformly distributed load per metre which the beam may carry, if the bending stress should not exceed 120 N/mm^2. Take $I = 8 \times 10^6\ mm^4$

Answer: Solved in this module.

Q.21. (AKTU 2018, 10 marks): A beam cross-section is used in two different orientations as shown in the given figure: Bending moments applied to the beam in both cases are same. Find the relation between the maximum bending stresses induced in cases (A) and (B)



Answer: $\sigma_A = 2\sigma_B$

Hint:

$$Z_A = \frac{b(b/2)^2}{6} = b^3/24$$

$$Z_A = \frac{(b/2)(b^2)}{6} = b^3/12$$

$$M = \sigma_A Z_A = \sigma_B Z_B$$

Q.22. (AKTU 2020, 10 marks): An I-section girder, 200 mm wide by 300 mm deep, with flange and web of thickness 20 mm is used as a simply supported beam over a span of 6 m. The girder carries a distributed load of 7 kN/m and a concentrated load of 23 kN at mid-span. Determine: (a) the second moment of area of the cross-section of the girder, (b) the maximum stress set-up.

Answer: $I = 186.4 \times 10^6\ mm^4$, $\sigma_{mt} = \sigma_{mc} = 53.11\ MN/m^2$

Q.23. (GTU 2020, 4 marks): A beam simple supported and carries an U.D.L. of 50 kN/m over whole span. The size of beam is 150 mm x 400 mm.. If maximum stress in the material of beam is 100 N/mm^2 find the span of beam.

Q.24. (UTU 2023, 10 marks): A timber beam of rectangular section is to support a load of 20 kN over a span of 4 m. If the depth of the section is to be twice the breadth, and the stress in the timber is not to exceed 60 N/mm, find the dimensions of the cross-section. How would you modify the cross-section of the beam if it were a concentrated load placed at the centre with the same ratio of breadth to depth.

Answer: Solved in this module.

Q.25. (PTU 2020, 5 marks): A beam 200 mm deep of symmetrical section has I = 8 x 10^7 mm^4 and is simply supported over a span of 8 m. Calculate (i) the UDL it may carry (ii) the concentrated load it may carry at the centre if maximum bending stress is not to exceed 100 N/mm^2

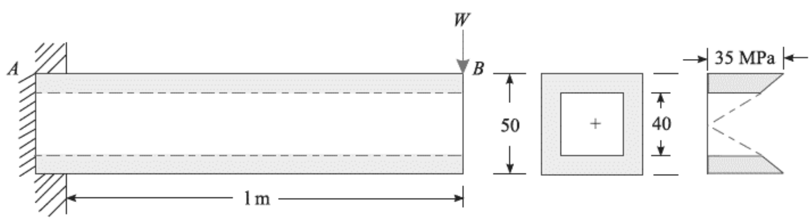
Q.26. (AKTU 2019, 10 marks): A water main of 500 mm internal diameter and 20 mm thick is running full. The water main is of cast iron and is supported at two points 10 m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3 respectively.

Answer: Solved in this module.

Q.27. (UTU 2014, 10 marks): A beam 500 mm deep of a symmetrical section has I = 10^8 mm^4 and is simply supported over a span of 10 m. Calculate UDL it may carry if the maximum bending stress is not to exceed 150 N/mm^2.

Answer: 4.8 kN/m^2

Q.28. (UTU 2022, 10 marks): A hollow square section with outer and inner dimensions of 50 mm and 40 mm respectively is used as a cantilever of span 1 m depicted in figure. How much concentrated load can be applied at the free end of the cantilever, if the maximum bending stress is not to exceed 35 MPa?

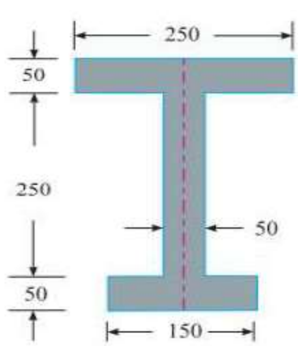


Answer: Solved in this module.

Q.29. (UTU 2015, 10 marks): A beam of length 5 m and of uniform rectangular section is simply supported at its ends. It carries a uniformly distributed load of 9 kN/m run over the entire length. Calculate the width and depth of the beam if permissible bending stress is 7 N, mm^2 and central deflection is not to exceed 1 cm.

Answer: 181.36 mm; 364.58 mm

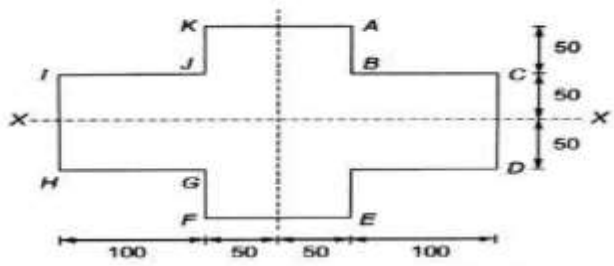
Q.30. (UTU 2022, 10 marks): A cast-iron bracket subjected to bending, has a cross-section of I-shape with unequal flanges as shown in figure.



If the compressive stress in top flange is not to exceed 17.5 MPa, what is the bending moment, the section can take? If the section is subjected to a shear force of 100 kN, draw the shear stress distribution over the depth of the section.

Answer: Solved in this module.

Q.31. (UTU 2022, 10 marks): A beam has cross-section shown in figure. It is subjected to a vertical shear force of 10 kN at a given section. Determine the shear stress distribution on the section.

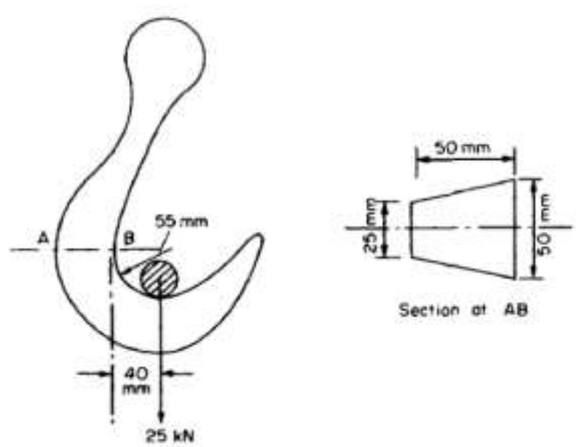


Q.32. (AKTU 2020, 2 marks): Differentiate between bending of straight and curved beam.

Q.33. (AKTU 2019, 10 marks): A curved bar of square section 4 cm sides and mean radius of curvature 5 cm is initially unstressed. If a bending moment of 300 Nm is applied to bar to straighten it, find the stresses at the inner and outer faces.

Answer: Similar problem is solved in this module.

Q.34. (AKTU 2020, 10 marks): A crane hook is constructed from trapezoidal cross-section material. At the critical section AB the dimensions are as shown in Fig. The hook supports a vertical load of 25 kN with a line of action 40 mm from B on the inside face. Calculate the values of the stresses at points A and B taking into account both bending and direct load effects across the section.



Answer: 129.2 MN/m^2; -80.3 MN/m^2

Similar problem is solved in this module.

Q.35. (AKTU 2018, 10 marks): Determine the location of neutral axis when a curved beam of trapezoidal section of bottom width 30 mm, top width 20 mm and height 40 mm is subjected to pure bending moment of +600 Nm. The bottom width is towards the center of curvature. The radius of curvature is 50 mm and beam is curved in a plane parallel to depth.

Answer: Solved in this module.